
# The Basics of Market Uncertainty: Price and Quantity are Complementary.

„Wise investors know their risks in advance. By examining a fundamental process of the economy – the determination of price and quantity – it is shown that relevant risks which market participants experience in advance are fundamentally different from the risks which afterwards statistically show up in the data. In particular it is demonstrated that in advance certainty about price and quantity are mutually exclusive."

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## A three-year-old, an art dealer and an aborigine make a market

When I was about three or four years old my uncle gave me a „Fünfliber" (5 Swiss francs). Very proud to be able to buy me a candy with my own money, I ordered one of the sweet „Fünfer-Bollen" (a piece of candy for five) from the saleslady in our village shop and gave her my „Fünfliber" for this – certain that a „Fünfer-Bollen" costs five francs. Very surprised that I got back a whole number of coins in addition to the candy, I looked at it with large and inquiring eyes. Laughing, she explained to me: The price is not five francs, but only five centimes.

This little story shows: The basic processes in the economy are actually very easy to understand. Nevertheless, difficulties can slip in. I would like to illustrate these difficulties. Raise questions. And give some initial answers. Probably quite surprising to you will be the proposal to use patterns of reasoning from modern quantum physics to explain a fundamental process of the economy – the determination of price and quantity. Don't worry; you do not need to be a physicist to follow these explanations. The basics from school are completely sufficient. But openness for novelty and for a different, unusual kind of reasoning can do no harm!

Let me briefly outline the subject of this essay by putting it into a broader context. Price and quantity are complementary. Complementary here means the following: One has two things, price and quantity, which together form a single whole. The single whole is the successful sales transaction. So far, everything seems clear and easy. There is no doubt about the size of the components. In retrospect, any transaction can be fully described by a distinct price and a distinct quantity. Price and quantity are risk-free in retrospect. This is the view of an accountant, or a statistician. Problems arise only if you don't look at the transaction afterwards (ex-post) but beforehand (ex-ante). Before and during a transaction, apparently, it is about finding the right price for the right quantity yet. But then price and quantity cannot be both known and determined in advance. There are necessarily, in advance, risks associated with the determination of price and quantity which no longer appear later in the data. Such ex-ante risks are important for market participants, even crucial. The art dealer asks for example: Do I get the Van Gogh or not? This is a big difference for her. Maybe, the existence of her museum depends on it. Her accountant does not care about such existential issues. She only asks: Is the price for the Van Gogh correctly entered in the books? The risk her client faces when acquiring the Van Gogh does not find its way in the books. An (economic) scientist, however, is likely to be interested in these risks. But, if these ex-ante risks do not show up in the data later, how can the scientist handle them? She uses quantitative, statistical models to represent risks. These models cannot see what is not in the data. In order to get the

appropriate data for these ex-ante risks she would have to go to the art dealer and ask her: Please try to buy the Van Gogh once again! Or – for the purpose of statistical significance – better a thousand times or, even better, a hundred thousand times again! Only with these data would she be completely confident her statistics representing these risks are right. She doesn't make a fool of herself and skips this "peculiar" request. Instead, she avoids any quantitative statement about these ex-ante risks and denotes the decision-making behavior of the art dealer as „qualitative". By „qualitative" she understands: „inaccessible" for the exact, empirical sciences which have to underlay their statements with data. Anyway: These ex-ante risks are doubtlessly important. Shouldn't the scientist therefore at least try to also „quantitatively" confine and model them? This essay seeks to answer exactly this question. (Other examples as the determination of price and quantity are not considered here. Nonetheless I would also like to pique your interest to address, with the very same question, other problems in the economy and the financial markets.)

We sum up briefly with regard to our subject theme. There are two sorts of „complements": complementary ex-post (after) and complementary ex-ante (before). Ex-post: Two things form a single whole – a distinct price and a distinct quantity result in a transaction. Ex-ante: The two things – price and quantity –, which both later form a single whole, are not yet defined in advance – the knowledge of price and the knowledge of quantity in advance are mutually exclusive.

Now let's put things into more practical terms and have a closer look at the determination of price and quantity. Let's start from the back, from the end result. Let us contemplate the successful transaction, as a whole. Can it be further partitioned? And if so, which is the smallest unit the whole consists of?

Imagine you are in a foreign country, lost and hungry. You meet an aborigine who has a fish in her hand. You would like to have the fish and point to it. The aborigine does not react. You think about what you could offer her in return. Of the ten coins you have you offer one in your open hand. No reaction. You put another coin in the offering hand. Again, no reaction. You continue adding coins until the aborigine – finally – to the eighth coin gives in and exchanges. As you can see, this is possible without anything else, in particular, without linguistic communication. The only thing that is required here is: Both players need to know what the offer of the other side is. The only element of communication in this coordination process is the adding (or not) of a further coin. More is not required. This language is understood worldwide: to add or to take away an additional coin. The coin, the monetary unit, is the smallest building block which makes a difference in this situation. The coin, the money, mirrors the whole. With this, we have already answered the question about the whole: The whole transaction can be split into steps which make a difference in the smallest monetary unit.

But why are, during these steps, the knowledge of price and the knowledge of quantity mutually exclusive? Usually, grownups know how many liters of milk they can purchase for 5.– in the supermarket around the corner. That the little three-year-old, who I used to be, was struggling with the price for his sweets isn't worth our while mentioning. Such irregularities do occur in efficient markets, but just by chance, with a small margin for error. Not more. However, if you look at this more closely, it may turn out to be less trivial. Couldn't it be that the situation of the three-year-old who doesn't know how many sweets he gets for his money is more of a rule than of an exception? Doesn't the grownup face a similar problem? She reckons that she will get three liters of milk for 5.– in the supermarket today. But how will she react suddenly realizing the higher prices tagged on the milk shelves? Or, if she learns after her shopping, that in the other supermarket across the street the same three liters of milk would have cost her only 4.–? That the price of 5.– has been only putatively certain for her?

And the saleslady in the village shop. Doesn't she have a similar problem? She puts a price tag on the shelf for her candies. But the other lady at the kiosk nearby could, without warning, lower her prices for candies. Therefore, she doesn't know how many candies the children, at this price, will buy in her shop, rather than buying at a possibly lower price, at a nearby kiosk.

How about the fisherwoman who has made a big catch? She asks herself: How will demand be today? Have other fishermen got similar big catches? Will this push the price? She doesn't know in advance the resulting price for her fish in the marketplace today.

These examples are chosen, such that the respective market participant of two things can determine or know only one thing in advance. Either price or quantity, but not both. Thus, the question indeed arises if this is always the case, whether knowledge of price and knowledge of quantity in advance are always exclusive.

## The basics of risk at an auction – and at other markets

Suppose you are an auctioneer for houses. You've got to auction a particular house. The minimum price the seller wants is 500'000.–. You are preparing for the auction. In the run-up to the auction you see various interested parties visiting the house. For most of them the house seems to be a very large investment. But there are some few of whom you think they may be very solvent. You ask yourself: How shall I run the auction? The minimum price is clear. Under 500'000.– you don't accept any bid. But how do you want to go on? Should you accept any higher bid? Couldn't it then be that the auction is staying a long time in the lower price area? Those to whom the house is a large investment may overbid each other by smallest amounts: 500'000.–, 501'000.–, 502'000.– … etc. The sluggish auction may give the impression that the house is not worth more. You risk losing the solvent buyers at an early stage. Or should you strongly filter out the auction, by accepting next bids at steps of at least 25'000.– higher only (500'000.–, 525'000.–,

550'000.– ... etc.). Making clear that you are particularly interested in the very solvent buyers? Here, you run the risk to lose the less solvent interested parties early. Bid dynamics may be hard going and the few solvent buyers may dictate the price to all the others.

In order to answer these questions you can try to put yourself in the shoes of the various potential buyers. For each bidder the uncertainty about the quantity to buy is the same: to own this house or not to own this house. We write for this measure of uncertainty $\Delta Q$:

$$\Delta Q = 1\ house - 0\ house$$

If I am a less solvent bidder, then I am interested in the possibility for small increases for next higher bids. I have got a budget of, say, 600'000.– at my disposal. If a fellow bidder before already has offered 599'000.– then I'm getting strongly in distress. I am reluctant to offer 600'000.–, my last bid, hoping nobody will overbid me anymore. The auction being at 599'000.– I'm taking an additional risk of 1'000.–. As soon as I have placed my bid, I'm again facing a new risk. The risk of being outbid by a fellow bidder at a price only slightly higher by, again, 1'000.– . We write for this *current price uncertainty* $\Delta p$ during the auction process:

$$\Delta p = \frac{601'000.-}{house} - \frac{600'000.-}{house} = \frac{1'000.-}{house}$$

Expressed in monetary units, my total current uncertainty is: the auction's minimal bid step. Or, more unexcitingly, expressed in math terms:

$$\Delta p \times \Delta Q = 1'000.-$$

This means: The current uncertainty about the price times the uncertainty about the quantity equals the minimal bid step. Again, as a less solvent bidder I appreciate the auctioneer's allowing for small bid steps in order to be able to bid with little effort. As a solvent bidder, on the other hand, I am interested to be able to outbid my competitors as soon as possible. For me, a few bid steps of 25'000.– may be quite doable. As soon as I'm in the highest bid then my competitors shall not have an easy play overbidding me. I love to attend auctions where, with each bid, bidders have to shoulder a relatively high additional cash uncertainty of

$$\Delta p \times \Delta Q = 25'000.-$$

In any case: The minimal bid step, that is the expression that stands on the right side of the equation (1'000.– in the example of an auction for less solvent bidders or 25'000.– at the auction for solvent bidders), denotes the minimum additional financial commitment a potential buyer has to put on the table if she wants to place a bid. Let us call this minimal bid step $h. Then we can write:

$$\Delta p \times \Delta Q = \$h$$

Each bidder, who is going to place her bid in the auction, faces a risk which is subject to this uncertainty equation:

*The current uncertainty about the price times the uncertainty about the quantity equals the minimal bid step.*

For bidders, in this situation, there is no absolute certainty, neither about price nor about quantity.

As auctioneer you define the conditions under which the auction is running. Under which trading takes place in this market. Thus, you determine the „constants of nature" in this market. You have put yourself in the shoes of the various potential buyers. Now you know which uncertainties potential buyers are facing. You are well aware of the objectives of the seller. Now you are in a better position to decide how to choose the „constant of nature" of the auction, how small or large bid steps shall be. Whatever you choose ($h=1.–, $h=1 000.–, or e.g. $h=25 000.–), the uncertainty equation for bidders applies! …

… that is, to be precise, the uncertainty equation does not always strictly apply. But in each market organization (with a fixed minimum quantum of money transaction $h) a so-called „market uncertainty relation" holds:

$$\Delta p \times \Delta Q \geq \$h$$

Here, I would like to spare you the math behind this subtlety ("≥" instead of "=") because the essential mechanics of the uncertainty in auctions and markets in general can already be understood with these basic calculations. (Nonetheless I hope, with this little essay, to whet your appetite to further develop this mathematics, this other reasoning. Especially for you, if you are among those readers who like to puzzle out tricky problems.) But what this mathematics shows is: If in the market the quantum of money transaction is fixed, then the certainty about the price can only be increased by reducing the certainty about the quantity and vice versa.

In order to illustrate this, consider this little example: As auctioneer you have already fixed the „constant of nature" for the auction, the minimal bid step, at e.g. 1'000.–. Prices are specified, for once, in monetary units per square meter of living space, for example 2'500.– per $m^2$. The house has, in total, living space of 200 $m^2$ with two apartments of 100 $m^2$ each. If you chose to auction the entire house then, for bidders, the uncertainty about the quantity is: to own the house (200 $m^2$) or not to own the house (0 $m^2$). The current price uncertainty at each bid step is 5.– per $m^2$. (If the total sales amount for the object has to increase by 1'000.– with each bid step then the price – for 200 $m^2$ living space – has to increase by at least 5.– per $m^2$, with auction prices of 2'500.–, 2'505.–, 2'510.– ... per $m^2$.) If you think that your potential buyers don't want to carry such a large quantity risk you can reduce it as follows: you divide the house into the two apartments of 100 $m^2$ and auction them separately.

Now, the quantity risk for bidders has decreased to: to own an apartment (100 $m^2$) or not to own an apartment (0 $m^2$). By keeping the bid step at 1'000.– (You're not allowing for change in your „constant of nature" of the auction) this is possible only with minimal price steps of at least 2 x 5.– or 10.– per $m^2$ living space, with auction prices 2'500.–, 2'510.–, 2'520.– ... per $m^2$. The current price risk has increased from 5.– to 10.– per $m^2$ living space. A reduced risk of quantity, thus, translates into a higher current price risk.

Let us sum up. Or, to put it in more precise terms, let us add together all the individual steps of our auction to a single whole – to a whole transaction. Let us string together all the steps. Starting from the minimum offer continuing to the first bid, to the second bid, to the third, the fourth …, and so on; stopping at the maximum bid, which finds itself unchallenged and thus awarded the contract. Which is the uncertainty for the whole transaction? The „small" uncertainties of each individual bid steps add up to a „big" uncertainty relation for the whole transaction. If I want to make sure that I will get the house (certainty about the quantity of house) I cannot set a personal price limit. I find myself in complete uncertainty about the amount of money I have to put aside for the purchase. If I want to limit the purchase price, that is to have price certainty, I cannot know whether I get the house at all. The result is an uncertainty about the quantity. I get the house: quantity 1, I don't get it: quantity 0.

The same thing which applies to buyers in auctions also applies to buyers in supermarkets. It applies to sellers. It applies to all possible systems of buyers and sellers in all possible market conditions. To the individual market participants and to the entire market as a whole:

*Certainty about price and certainty about quantity in advance are mutually exclusive.*

Only, by our everyday shopping in the supermarket, it seems to us as if we could have certainty about both at the same time in advance. The truth is in fact that we, in our daily lives too, know very little about the conditions of the supermarket, the retail market, or any market at all. We do not know whether in the store next door a yet cheaper deal is waiting for us. Or whether it is worth our while to better wait for the purchase of a planned major acquisition. Never can we know all, for us, relevant conditions of the market. We can look at the market from any point of view we like. Never can it be that everything is defined or known. This, after all, is the very reason why we are talking about a „market", i.e. about a place of free – and therefore not predefined – exchange and not about a „forced exchange mechanism". It is always possible that we are, to a limited degree, able to predefine certain things as market participants or to know certain things in advance as market observers: for example either know in advance the exact price or predefine the exact quantity, or know in advance a little of the price and a little of the quantity – but the remaining rest can neither be determined nor known by us. The rest, the "complement" always determines the market out there – the part of the market which always remains unknown to us.

## The edification which lies in the fact that God may be playing dice

If you remember your school physics doesn't this uncertainly relation sound somewhat familiar to you? A similar statement can be made about the smallest physical particles. If we just measure the location of such a tiny particle we cannot measure, at the same time, its momentum. If we want to find out something about the movement of the particle we lose the knowledge about the whereabouts. We can determine only one of both. Or we find out a small bit about both and then we know „more or less" where it is and „more or less" where it moves to. In observing the smallest particles of matter some degree of uncertainty is so natural. There's even a formula exactly bordering this necessary measure of uncertainty: the Heisenberg uncertainty principle. The fact that there is a minimal quantum of action h, the minimal „transactional unit of nature", also called Planck's constant, causes an inequality, quite similar to our „market uncertainty relation", to hold:

$$\Delta x \times \Delta p \geq h$$

The uncertainty about position ($\Delta x$) and momentum ($\Delta p$) cannot be eliminated because of this very same uncertainty relation. If we take this uncertainty relation seriously (and that we must at the current state of physics) then our common understanding of cause and effect gets, mildly spoken, rather challenged. An indefinite position. What is that? If we measure it the result, by chance, is random. But should chance, randomness, really be an element in the realm of science? We do know chance, randomness, very well, but from our everyday lives only. Chance, for us, is everything that happens what we don't anticipate. For example, that by chance we run into uncle Fridolin buying milk in the supermarket. Isn't the very role of science in fact to explain everything we, as ordinary people, cannot explain? In the realm of science there really shouldn't be anything like randomness or chance, since in reality everything has to have its cause, hasn't it? And, again, isn't the very role of science in fact to identify these causes and explaining them to us?

The two physicists Niels Bohr and Albert Einstein got into a veritable battle of words, some eighty, ninety years ago, on exactly this issue. After a hefty dispute, Einstein is supposed to have said: „God doesn't play dice with the universe!" As a scientist, he could not accept that randomness should rule the world. He admitted, though: The world may appear to us, due to quantum mechanics, random. But in the background the world is obeying strict laws of cause and effect.

Let us transfer this dispute to the economy, to bourses and markets: If I want to buy a house, for example, then I can either assume, with Einstein, that the outcome of the auction is essentially predetermined. Because well-defined, but to me hidden laws of cause and effect are doing their work in the background. Or I can reason, with Bohr, that a certain amount of fundamental randomness determines the auction. In both cases I am subject to the "market uncertainty relation", the basics of market uncertainty.

I don't want to set myself up as an arbiter between these two ways of thinking, and certainly not as an arbiter between these two giants of physics. Something new, however, entered with the discovery of quantum mechanics the realm of science: the observer. The observer of physical phenomena has a much stronger role than previously assumed. The observer – represented in the measuring apparatus – affects the measurement itself. The result of the measurement appears (Einstein) – or just becomes (Bohr) – random.

Aren't we, as market participants, exactly in the same situation as the observer in physics? Don't we affect, as market participants, decisively our (measuring) result? Isn't randomness created (doesn't it appear) exactly because of this? Because we ourselves participate actively in the economic process, participate in the financial markets? Is it not our very own participation which makes the economic act so unique? Which makes it so difficult – often impossible – for scientists to repeat this unique economic act for their statistical purposes? And is not the important difference between the risk before (ex-ante) and the risk after (ex-post) created precisely because our involvement with the economic process? Whether we like it or not, we cannot escape the basics of market uncertainty. But with the basics of market uncertainty we can quantitatively confine this forward-looking risk. This ominous ex-ante risk which does not show up later in the data. But honestly: How can we seriously wish to escape this market uncertainty? Is it not this very uncertainty which gives weight to our economic decision? Which gives us, therefore, responsibility? And which gives our decision, therewith, meaning?

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