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What is the fair price for information in financial markets?

Outline of a realistic aggregate capital markets theory explaining bubbles and trading away from equilibrium by means of Maxwell's theory of electromagnetism

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Abstract

In this essay, we give an introduction into a new kind of thinking about how information is processed in financial markets and how this relates to returns as prices for information and prices for time, and as prices for risk. It turns out that complex phenomena like trading away from equilibrium or the acceleration impact of value trading on price movements can be modeled using linear physical theories such as Maxwell's theory of electromagnetism or quantum mechanics. For some basic market situations of information processing complete mathematical proofs are given. We formulate an agenda and a series of conjectures on how these basic ideas could be used to establish a so called "realistic capital markets theory", an RCMT.

Key words

Capital markets theory, information processing, electromagnetism, quantum mechanics.

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1. Introduction

The main purpose of this article is to establish a basic methodology to use *linear* concepts of physics, in particular Maxwell's theory of electromagnetism, to formally deal with a list of complex phenomena in financial markets, like bubbles or trading away from equilibrium, and to outline a realistic capital markets theory addressing these complex phenomena. These phenomena could, by now and the means of classical methods of financial and economic equilibrium theories, neither been understood nor described appropriately. In the last ten to twenty-five years, however, a new movement, called *econophysics*, in the scientific field of economics and finance has emerged trying to cope with complex phenomena, like non-linear dynamics, power-laws and chaotic behavior of economic systems, with non-classical methods (for an overview see (Mantegna, et al., 2000), single contributions like (Arthur, 1994), (Peters, 1996), (Mandelbrot, et al., 2004) caused a broader and intense scientific discussion about the topic). This article can be viewed as a contribution to these *econophysical* attempts. What differentiates the methods in consideration here with other contributions to *econophysics* is the *sole* use of *linear* physical theories in order to understand and explain complex economic phenomena such as bubbles and trading away from equilibrium, phenomena which have been normally viewed as highly *non-linear* supposed to be treated and understood exclusively using *non-linear* physical and mathematical modeling. To mark an exception, there has been an increased use of quantum mechanical techniques to solve problems in economics (see (Baaquie, 2004) or (Kondratenko, 2005) and others), quantum mechanics being a *linear* theory able to cope with a least some complex phenomena such as uncertainty stemming from complementarity of position and momentum or the problem of the interconnection between the observer and the observed system. What characterizes these attempts, especially in (Baaquie, 2004) and in other contributions, is that they try to solve classically derived equations, e.g. for option pricing, with the non-classical methods of quantum mechanics; in other words: they look at economic problems with the eyes of classical equilibrium theories, but try to solve these problems with non-classical-models (quantum mechanics) not procreating a new, or additional, understanding of the underlying economic phenomenon. Here we use physics not to solve already existing economic equations but to establish new equations and a capital markets theory, which are *linear* and therefore hopefully subject to easy to understand new insights in the underlying economics.
Hence, this article seeks to give a basic idea how *linear* physical modeling can be used for the understanding of complex capital markets. Economic versions of Maxwell's equations of electromagnetism are formally derived for the explanation of price-movements away from a given economic equilibrium (*section 5*).
The special role of *information*, i.e. what is known in a system, what is measured or not, is both seen, in light of an economic situation of the market and in light of the electromagnetic wave or photon. In quantum physics, for example, a light particle, i.e. a photon, can only bear *information* about momentum and energy, not about position. If position of a photon is measured it is completely random. Similar effects can be observed if we try to model *information* in a financial market context, namely if we distinguish between *un-informed* and *informed* investors and try to understand their impact on financial markets. For example, a group of investors can have distinct *information* about the equilibrium price around which a market price can oscillate, but have no information about their relative position in the participation cycle which causes the oscillation.
To the end of understanding some basic insights of *information* in financial markets we restrict ourselves in a first step to two simplified economic situations:
a) where all market participants, called the natural buy-and-hold-investors, are *neither* informed about the true equilibrium price *nor* about their relative position within the participation cycle (*section* 2) and
b) where there are two groups of investors, one without any knowledge about equilibrium price and relative position, the aforementioned natural buy-and-hold-investors, and one group of investors, the value-traders, who have a special insight in the true equilibrium price, but also not on their relative position in the participation cycle *(section 4)*.
One of the key findings of this article is that, from the point of view of financial markets, price for information should be viewed as the profit and loss of the value-traders *(section 6)*. This price for information is fair in a risk neutral world, if and only if the acceleration effect, the value traders have on the achievement of market equilibrium, is positive and comparable to their profit.
In *section* 7 an outline of an aggregate capital markets theory is given where price fluctuations are related and analytically explained through a one-to-one correspondence with a series of fundamental changes in natural market. A classification of all possible investment strategies in this context is given. Using an adequate equilibrium argument, the main result will be stated as follows *(section 7)*: Market is in "fair equilibrium between the different investment strategies" if and only if

Price for information = price for time = price for risk.

Section 8 shall give a series of applications of practical and theoretical use of the derived capital markets theory. In particular, we try to understand convergence towards equilibrium and the formation and bursting of bubbles. In the last *section 9* we give an outlook for extension of usage of linear concepts, as theory transfer from electromagnetism, derived here to model additional market effects not addressed in this article, namely the application to single security information processing, and extension to theory transfer from quantum mechanics.

2. The natural market with non-informed investors and the buy-and-hold-code

In classical equilibrium theories a single type of market participants is introduced, the *rational homo oeconomicus*. A direct consequence of the introduction of this single type of investors, besides all other conditions of the classical theory (efficient market hypothesis, homogeneous expectations, etc.) is that there is *no trading* in such markets by the lack of counterparties (Ingersoll, 1987, Theorem 1, p.76). Since we want to investigate phenomena like *trading* away from equilibrium, we have to find another, more realistic, basic disposition of a financial market.
For this end, let us introduce and define the *natural investor* in the financial market.

Definition 1. The natural investor and the buy-and-hold-code.

- *The natural investor is defined to be an investor who uses a buy-and-hold-code for her buy and sell orders in the financial market.*
- ***The buy-and-hold-code** is defined by the following instructions to (of) the investor:*
    - *if you have money saved then buy financial assets at any given market price*
    - *if choose to consume some of your financial wealth saved then sell financial assets at any given market price to the extend you've chosen to consume*
    - *Stop*
- *The natural investor is non-informed in the sense that her market behavior is independent from any given market price. In particular, she has no opinion on whether a security is cheap or expensive at a given price.*

This definition of the natural investors allows us to define a *natural turnover* in trading and other relevant quantities within the financial market under consideration.

Definition 2. Natural turnover, time, price adaptability and demand-supply-overhang.

- *A natural market, i.e. a market of financial assets with natural investors only, is said to be in constant equilibrium at price p if, at p, a natural turnover (volume) $\dot{V}_0 = \dot{V}_{D,0} = \dot{V}_{S,0}$ is traded per unit of time, and natural demand $\dot{V}_{D,0}$ meets constantly natural supply $\dot{V}_{S,0}$ of tradable securities[1].*

- *Time t is measured in units of aggregate market activity. If market has a general bookkeeping function, t is measured in units of general ledger entries.*

- *If in sum buy and sell orders of the natural investors are not in equilibrium for the market price p then the imbalance will be reduced by the following price adaptability equation expressed in*

Assumption 1. $\dot{p} = f \cdot O_{D/S} := f \cdot \int (\dot{V}_{D,0} - \dot{V}_{S,0}) dt,$

Where $f > 0$ is called the price adaptability and $O_{D/S}$ the demand-supply-overhang resulting from the aggregation of imbalances of buy and sell orders of investors.

A first and basic consequence of *Assumption 1* is that the market will not immediately be able to clear every demand-supply imbalance, but that this process takes time according to the clearing mechanism and capacity of the market. In this sense the price adaptability f can be viewed as clearing capacity of the market in relation to the amount of aggregate un-cleared buy and sell orders of investors.

If natural market is in equilibrium and demand meets supply constantly then a constant trading volume $\dot{V}_0$ at equilibrium price p_0 is observed.

Theorem 1. Natural equilibrium in natural market.
In equilibrium of natural market, the cash savings per unit of time equals the stock supply per unit of time times equilibrium price p_0:

$$\dot{C}_D = \dot{V}_{S,0} \cdot p_0,$$

where $\dot{C}_D$ denotes the savings (rate) per unit of time, creating natural investor demand.
Theorem 1 is a trivial consequence of the natural definitions above.■

It raises the questions: what happens if this equilibrium condition is hurt?

3. Introducing pre-information: fundamental change in natural market

We still consider pure natural market situations, with natural participants only, not knowing anything relevant about the market except their own buy and sell orders. Therefore, we cannot speak about information, in the usual sense, if some relevant change in the market takes place since nobody will be informed on this relevant change. Let us call such an event *pre-information*

emphasizing the fact that a relevant change has occurred in the market place *without* conscious awareness of the market participants.
In light of the equilibrium situation of Theorem 1 the simplest event of change we can think of would be a sustained demand-supply imbalance of natural buy and sell orders

Example 1. Savings shock in natural market.

Positive savings shock of natural investors is defined as follows:

For times $t < t_{event}$: $\dot{C}_D = \dot{C}_0 = \dot{V}_{S,0} \cdot p_0$, *and*

for times $t \geq t_{event}$: $\dot{C}_D = \dot{C}_1 = \dot{V}_{S,0} \cdot p_1 > \dot{V}_{S,0} \cdot p_0$,

where p_1 *(and* p_0*) denote the new (and old) equilibrium price,*

while natural supply remains constant: $\dot{V}_{S,1} = \dot{V}_{S,0} = \dot{V}_0 = const.$

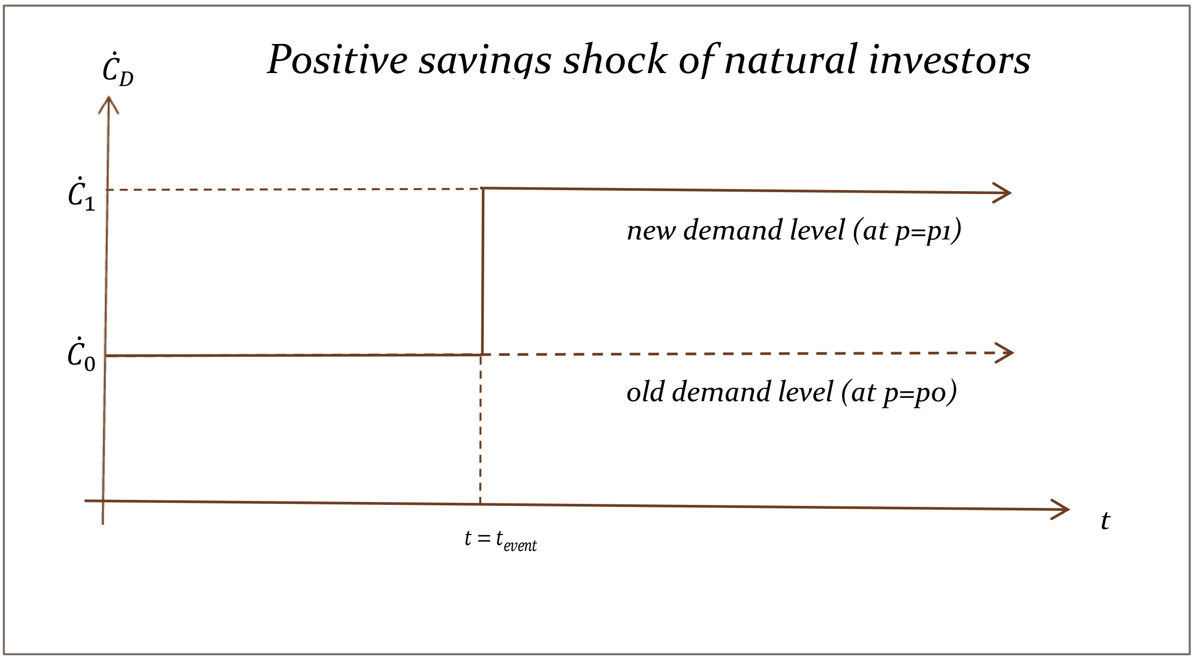


Diagram 1.

Remark. Note that such an immediate and sustained increase in natural savings can occur without conscious awareness of the natural investor. The natural investor may view the increased amount of money saved as a one-off-effect which will not repeat itself in future, whereas the underlying economic situation could be fundamentally and sustainably have changed for her to the better. In this article we define a *fundamental change* in the market as a sustainable change of the relation between natural demand and supply. Hence, the needs and the solvency of natural investors determine the fundamental equilibrium price of tradable securities, not the return expectations of these securities themselves directly. Return expectations of tradable securities can (later after their introduction to the market) be made by informed investors, such as value traders, but the criterion of whether their expectations are right or wrong is whether these expectations correctly predict the effects of future change with regard to natural demand and supply. Example 1 describes a fundamental demand shock caused by increased savings of natural investors, calculations become easier, without changing the quality of the results, if we modify *Assumption 1*, a little. Instead of using the natural demand-supply-overhang $O_{D/S}$ we use the cash-equivalent of $O_{D/S}$:

$$O_{D/S}^{Cash}(p_0) = \int_{t>t_{event}} (\dot{C}_1 - \dot{C}_0)dt = \int_{t>t_{event}} \dot{V}_0(p_1 - p_0)dt,$$

here in the case where, after the savings shock, the market would constantly trade at the old equilibrium price p_0. In order to define price adaptability of the market, we let the trading price p float ($p_0 \to p$)and use $O_{D/S}^{Cash}(p)$ accordingly to modify *Assumption 1* to

Assumption 1*: $\dot{p} = f \cdot O_{D/S}^{Cash}(p)$, where

$$O_{D/S}^{Cash}(p) := \int_{t>t_{event}} \dot{V}_0(p_1 - p)dt.$$

The dynamics of the price movement of a fundamental change as illustrated in Example 1 (sudden increase of natural savings causing a sustainable demand shock) can now be completely mathematically described.

Theorem 2. Harmonic market price oscillations caused by pre-information in natural market.
Given a pure market of natural investors with the following pre-information: At times $t < t_{event}$ *market is in equilibrium at price* p_0 *with constant natural turnover* $\dot{V}_0$ *per unit of time, and, in addition, at time* $t \geq t_{event}$ *a sustainable savings increase occurs*

(according to Example 1 with new equilibrium price p_1. If natural supply $\dot{V}_{S,0} = \dot{V}_0$ is kept constant then the market price oscillates harmonically around the new equilibrium price p_1, with frequency $\omega_0 = (f \cdot \dot{V}_0)^{1/2}$ and amplitude $A_0 = (p_1 - p_0)$.

Proof. Define the relative market price to the new equilibrium as $\Delta p := p - p_1$. Since $\dot{V}_{S,0} = \dot{V}_0$ is assumed to be constant with respect to *t*, differentiation of price adaptability equation in Assumption 1* with respect to *t* yields the well-known differential equation for harmonic oscillations

$$\Delta \ddot{p} = -f \cdot \dot{V}_0 \cdot \Delta p.$$

The solution of this differential equation, satisfying the initial condition $\Delta p(t_{event}) = (p_0 - p_1)$, is given by

$$\Delta p = (p_0 - p_1) \cdot \cos(\omega_0 \cdot (t - t_{event})),$$

with frequency $\omega_0 = (f \cdot \dot{V}_0)^{1/2}$ and amplitude $A_0 = (p_1 - p_0)$. ■

A first trivial consequence of Theorem 2 can be stated as follows.

Lemma 1. Turning point lemma for natural market.
Under the assumptions of Theorem 2 the market reaches extreme price levels if and only if natural market is cleared, i.e. natural participants in sum have completed transaction of their buy and sell orders and there is zero demand-supply-overhang.

Proof. Market reaches extreme levels if and only *if* $\Delta \dot{p} = 0$, which, by Assumption 1*, is given if aggregate demand-supply-overhang vanishes. ■

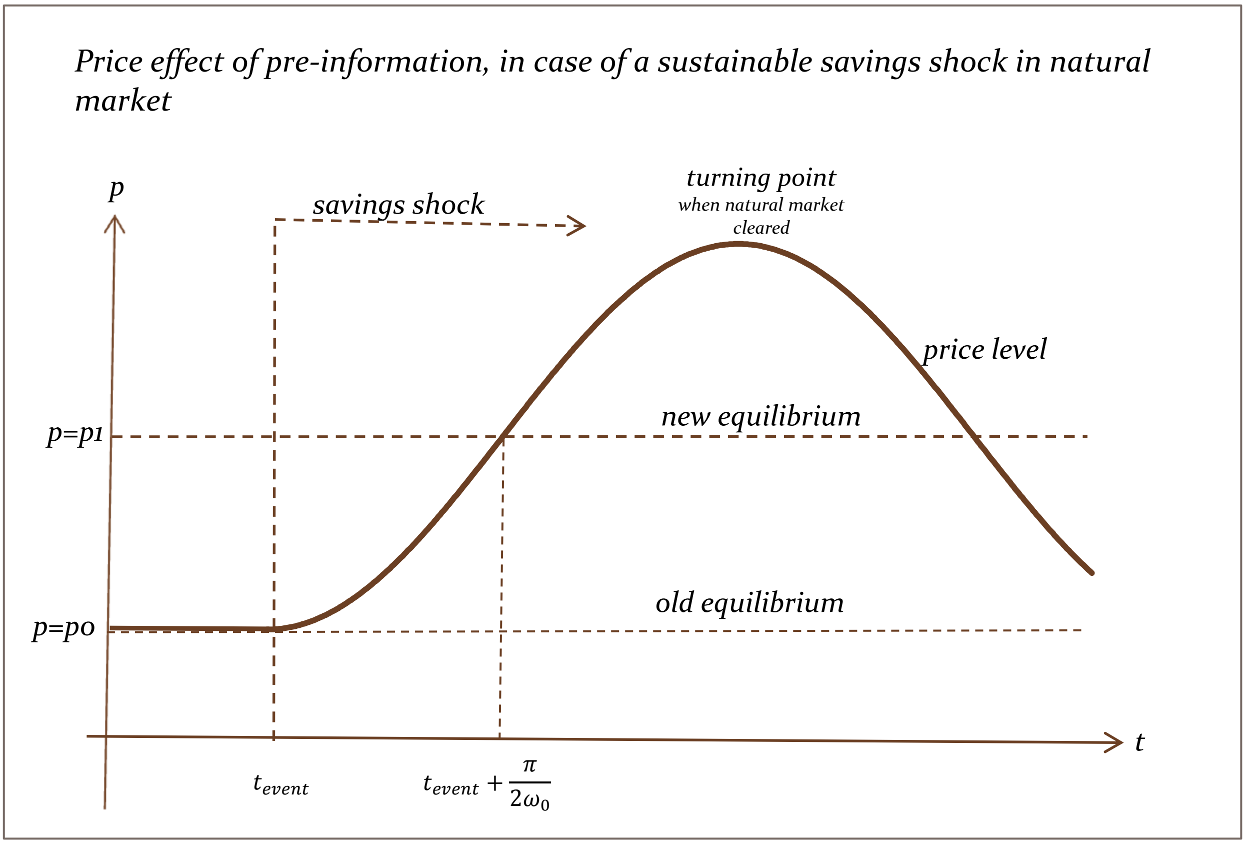


Diagram 2.

4. Introducing information: a group of informed value investors start to trade in natural market

Up until now we have assumed that none of the investors knows about a given fundamental change in market like a sustainable savings shock as described in Example 1. Now let us assume, in addition to a given population of natural market participants, that there is a group of investors able to analyze and observe such an fundamental change, in the market among natural investors, from an old equilibrium price p_0 to a new equilibrium price p_1.

Definition 3. The informed value trader and the value code.

A (n idealized) informed value trader is defined as an investor

a) *who holds securities only short-term, i.e. if $\dot{V}_{value}$ is her trading volume in stock securities per unit of time then she is not accumulating stock over time: $\int_{t>0} \dot{V}_{value} dt$ is bounded around zero,*

b) *who is informed about the correct theoretical natural equilibrium price* p_1 *and*
c) *whose trading behavior can be described by the following equation:*

Assumption 2. $\dot{V}_{value}(p) = \alpha \cdot \dot{V}_0 \cdot \frac{(h(p_1)-p)}{p}$,

which gives rise for the following ***value-code****:*

- *observe price p in market at time t*
- *submit buy or sell orders to market in order to ensure purchase or sales of* $\alpha \cdot \dot{V}_0 \cdot \frac{(h(p_1)-p)}{p} \cdot \Delta t$ units of securities by time $t + \Delta t$
- *stop*

The parameter α *characterizes both the value trader's ability* <u>*and*</u> *intention to crowd out natural demand or supply at a given price level p. We therefore call* α *the value trader's crowding-out power.* $h(p_1)$ *is called the calculatory fair value of securities traded. In the simplest case, the value trader uses is* $h(p_1) = p_1$ *for her trading activities. But, as we will see later in section 6, it is optimal for a value trader to use a calculatory fair value of* $h(p_1) < p_1$ *in order to ensure that she will not accumulate stock over time.*

We now have two groups of investors in the market place, the natural investors, who, by Assumption 1*, can accumulate demand-supply overhang if market price is not trading at natural equilibrium, and the value traders, who do not accumulate a demand-supply overhang of their buy and sell orders, since by Definition 3 and their crowding-out power they are always capable to complete their transactions in the market immediately. Since value traders are able to crowd-out natural demand or supply, this crowded-out demand (or supply) adds up to the already existing (old) demand-supply overhang of natural investors. Assumption 1* now reads

Assumption 1**: $\dot{p} = f \cdot O_{D/S}^{Cash}(p)$, where

$$O_{D/S}^{Cash}(p) = O_{D/S,\,natural\text{-}old}^{Cash}(p) + O_{D/S,crowded\text{-}out}^{Cash}(p) =$$

$$= \int_{t>t_{event}} \dot{V}_0(p_1 - p)dt + \int_{t>t_{event}} \alpha \cdot \dot{V}_0 \cdot (h(p_1) - p)dt.$$ [2]

Again, as in natural market of section 3, we can fully describe the dynamics of the price movement of a fundamental change as illustrated in Example 1 (sudden increase of natural savings causing a sustainable demand shock) also if market consists both of non-informed natural investors and informed value traders.

Theorem 3. Accelerated harmonic market price oscillations in partially informed market.

Given a pure market of natural investors with the following pre-information: At times $t < t_{event}$ *market is in equilibrium at price* p_0 *with constant natural turnover* $\dot{V}_0$ *per unit of time, and, in addition, at time* $t \geq t_{event}$ *a sustainable savings increase occurs (according to Example 1) with theoretical new equilibrium price* p_1*.*

Let now, for times $\geq t_{event}$ *, informed value traders, with aggregate crowding-out power* α*, join natural market forming a partially informed market.*

If natural supply $\dot{V}_{S,0} = \dot{V}_0$ *is kept constant, then the market price oscillates harmonically around a new real equilibrium price* $g(p_1) := \frac{p_1 + \alpha h(p_1)}{1+\alpha}$*, which is close but below the theoretical new equilibrium price* p_1*, with frequency*

$$\omega_1 = ((1+\alpha) \cdot f \cdot \dot{V}_0)^{1/2} \text{ and amplitude } A_1 = (g(p_1) - p_0).$$

We observe that the frequency $\omega_1 = (1+\alpha)^{1/2} \cdot \omega_0$. Therefore value traders with aggregate crowding-out power α accelerate price oscillations caused by pre-information by the factor $(1+\alpha)^{1/2}$ in comparison to pure natural market.

Proof. Define the relative market price to the, yet unknown, new real equilibrium price: $\Delta p := p - g(p_1)$. Since $\dot{V}_{S,0} = \dot{V}_0$ is assumed to be constant with respect to *t*, differentiation of price adaptability equation in Assumption 1** with respect to *t* yields the following differential equation

$$\ddot{p} = -f \cdot \dot{V}_0(p_1 - p) - \alpha \cdot f \cdot \dot{V}_0(h(p_1) - p).$$

This differential equation can, with some basic algebra, be transformed into harmonic form

$$\Delta\ddot{p} = -f \cdot (1+\alpha) \cdot \dot{V}_0 \cdot \Delta p,$$

with $\Delta p = p - g(p_1)$ and $g(p_1) := \frac{p_1 + \alpha h(p_1)}{1+\alpha}$.

Again, the solution of this differential equation, satisfying the initial condition $\Delta p(t_{event}) = (p_0 - g(p_1))$, is given by

$$\Delta p = (p_0 - g(p_1)) \cdot \cos(\omega_1 \cdot (t - t_{event})),$$

with frequency $\omega_1 = ((1+\alpha) \cdot f \cdot \dot{V}_0)^{1/2}$ and amplitude $A_1 = (g(p_1) - p_0)$. ■

Again, as a first trivial consequence of Theorem 3 the turning point lemma still holds true.

Lemma 2. Turning point lemma for partially informed market.
Also under the extended assumptions of Theorem 3 the partially informed market reaches extreme price levels if and only if natural market is cleared, i.e. natural participants in sum have completed transaction of their buy and sell orders and there is zero demand-supply-overhang.

Proof. Market reaches extreme levels if and only if $\Delta\dot{p} = 0$, which, by Assumption 1**, is given if aggregate natural demand-supply-overhang vanishes. ■

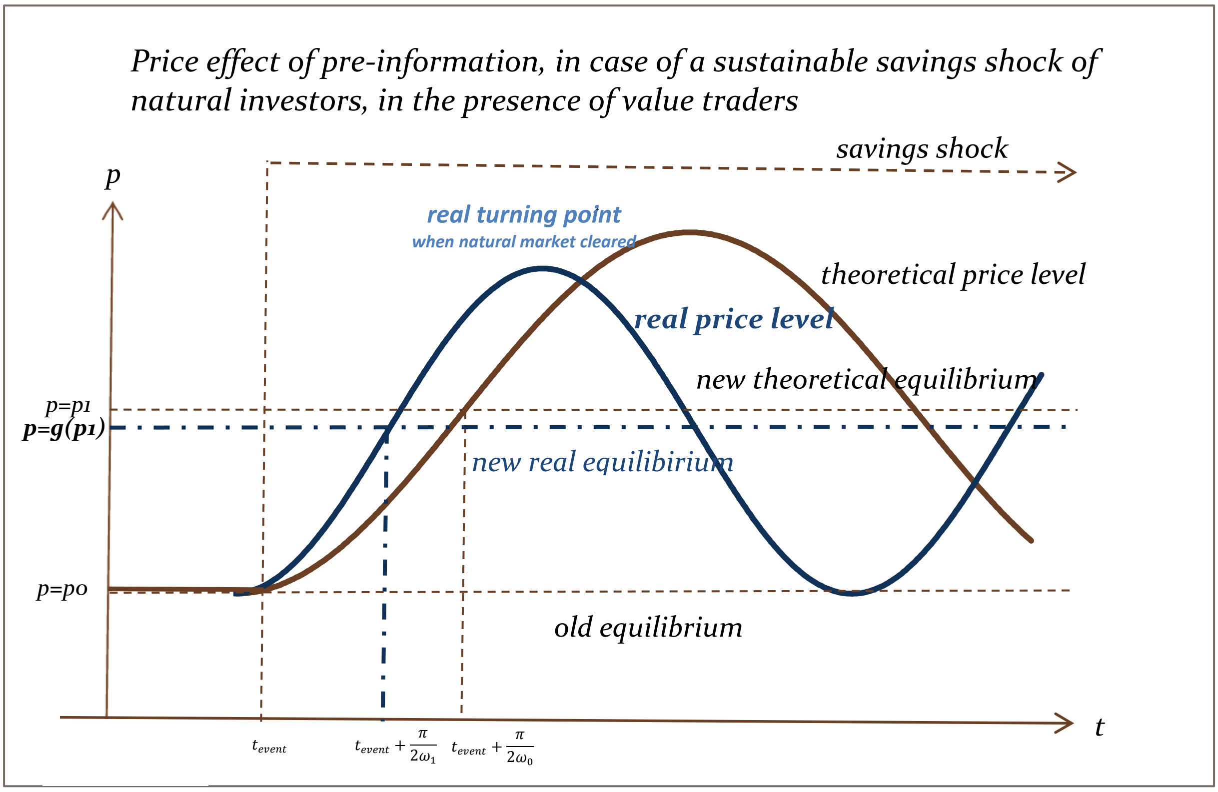


Diagram 3

5. Using Maxwell's equations of electromagnetic waves to describe natural, and partially informed, market effects: formally understand price oscillations and trading away from equilibrium

In the last two sections we have shown how harmonic price fluctuations around a given equilibrium can occur if a fundamental change, as the pre-information event of Example 1, happens both in a pure uniformed natural market and in a partially informed market. Since we plan to use linear physical theories in order to gain further understanding of economic phenomena like bubbles and trading away from equilibrium it is obvious to watch out for harmonic oscillations in physical modeling. There are many candidates for this: the mathematical pendulum, string motions, or many other motions of physical waves. One physical theory of harmonic oscillation distinctly stands out, Maxwell's theory of electromagnetism, since it lays the foundation for the more advanced theory of quantum mechanics, which in turn can be viewed as a theory of information (Bouwmeester, et al., 2001). Thus, while transferring Maxwell's theory of electromagnetism we also intend to lay a foundation for a theory transfer of quantum information theory to financial markets.

But, how could such a theory transfer of the Maxwell's electromagnetism be accomplished? First, and mainly, we need to know what space and time in the situation of natural or partially informed markets of the last sections may be.

Definition 4. Three-space and time.

- *X-dimension is spanned by the market collection and distribution function (MCD-code):*

  ***MCD-code (Market collection and re-distribution code)**:*
  - *collect all buy and sell orders (cash and securities) from market participants*
  - *submit orders (cash and securities) to MSC function (market storage and clearing function see below)*
  - *receive closed transactions from MSC function*

- ➢ ***re-distribute*** *transacted securities to market participants*
- ➢ *stop*

Call $x - axis$ *= coordinate spanned by MCD-code.*
Normalize Δx *= 1 after 1 run through 1 MCD-code.*

- ***Y-dimension*** *is spanned by the market storage and clearing function (MSC-code).*

MSC-code (Market storage and clearing code)*:*
- ➢ *receive orders (cash and securities) from MCD function*
- ➢ ***store*** *cash and securities in central security store*
- ➢ ***clear*** *demand and supply at current price p*
- ➢ *send transacted securities back to MCD function*
- ➢ *leave un-transacted securities in central security store as net overhang* $O_{D/S}^{Cash}$
- ➢ *increase current price by* $\Delta p = f \cdot O_{D/S}^{Cash} \cdot \Delta t$ *for next clearing round*
- ➢ *stop*

Call $y - axis$ *= coordinate spanned by MSC-code.*
Normalize Δy *= 1 after 1 run through 1 MSC-code.*

- ***Z-dimension*** *is spanned by the emergent market participation function (MP-code).*

MP-code (Market participation code)*:*

- ➢ *take an investor group (natural investors or value traders)*
- ➢ ***determine phase*** θ *of the cyclical motion of their market participation (according to Theorem 2 or Theorem 3)*
- ➢ *let time t run through 1 unit of* Δt
- ➢ ***determine phase*** $\theta + \Delta\theta$ *of the cyclical motion of their market participation (according to Theorem 2 or Theorem 3)*
- ➢ *stop*

Call $z - axis$ *= coordinate spanned by MP-code.*
Normalize Δz *= 1 after 1 run through* $\Delta\theta$*.*

- ***Time t*** *is spanned by the market's general ledger code (MGL-code).*
According to Definition 2, time t is measured in units of aggregate market activity. If market has a general bookkeeping function, t is measured in units of general ledger entries.
Here, in particular, the pace of t, the bookkeeping activity, shall be synchronized with above market activities, such that $\Delta x = \Delta y = \Delta z = \Delta t$.

MGL-code (Market general ledger code):

- ➢ *update collection and re-distribution of cash and securities of MCD function in market's general ledger after one run through MCD-code*
- ➢ *update store and clearing of cash and securities of MSC function in market's general ledger after one run through MSC-code*
- ➢ *stop*

Call $t - axis$ *= coordinate spanned by MGL-code.*
Normalize Δt *= 1 after 1 run through 1 MGL-code.*

At first sight, the definition of three-space and time might look somewhat artificial. In a sense this is intentionally so: we are shaping the economic space and time in a way that they are prepared to comply naturally the linear physical theory of Maxwell's electromagnetism. On the other hand the definitions make sense also from an economic point of view. A closer look at the four different codes (MCD-code, MSC-code, MP-code and MGL-code) confirms the assumption that they reflect independent market activities giving rise for a four dimensional orthogonal spacetime *(x, y, z, t)*. We highlight a special property of the market participation code. The MP-code does not need a separate market entity to perform its activity. The MP-code just happens. It is "emergent" along with MCD and MSC. Nonetheless is it independent from MCD and MSC, since the participation cycle, by Theorem 2 and Theorem 3, emerges for any value of market adaptability $f > \mathbf{0}$ and any possible realization of MCD and MSC. We also emphasize that the speed *c* of all activities, which are by definition well-synchronized, is the same and standardized to 1:

$$\frac{dx}{dt} = \frac{dy}{dt} = \frac{dy}{dt} = c = 1,$$

giving us a first reference to the constant speed of light of electromagnetic waves. We also point out to the fact that we have introduced several quantities, into market, which are not disclosed to market participants, namely phase of participation cycle θ, price adaptability f or demand-supply overhang $O_{D/S}^{Cash}(p)$. These are "hidden variables" to market participants. If they were disclosed, market participants would be able to exploit these additional pieces of information, which in turn would disturb the system to an extend which could not be described anymore by classical physical theory. An outlook of the impact of (partial) disclosure of these quantities to market participants is given in *section* 9.

Before we will be able to derive Maxwell's equations, we need to define electric and magnetic field in our market context.

Definition 5. Electric and magnetic field in context of natural and partially informed market.

- *Electric field* $\vec{E} := \{E_x, 0, 0\}$, *where*

  E_x *:= total net collection by MDC function =*

  *={(securities for sale) * p – cash for purchase of securities}*

- *Magnetic field* $\vec{H} := \{0, H_y, 0\}$, *where*

  H_y ≔ *Total net storage by MSC function* $= O_{D/S}^{Cash}(p)$.

With this definition it follows immediately that change of a given electric field always goes along with the creation of a magnetic field and vice versa, as we would expect from classical electromagnetism.
Now we are able to formulate one of the basic results of this article.

Theorem 4. Maxwell equations of electromagnetism in natural and partially informed market.

Given the same assumptions as in Theorem 3 (or of Theorem 2 in case of $\alpha = 0$) with the pre-information event of Example 1. Let, in addition, spacetime (x, y, z, t) be spanned up by the market activities according to Definition 4 and balances of supply-demand overhang be defined according to Definition 5, as represented by the net entries in general ledger of the two market functions (collection-distribution and securities-storage-and-clearing).
Then the pre-information event of Example 1 causes the creation of an electromagnetic wave in spacetime (x, y, z, t) fulfilling the four Maxwell equations:

1. $rot\,\vec{H} = \frac{\partial}{\partial t}\vec{E}$
2. $rot\,\vec{E} = -\frac{\partial}{\partial t}\vec{H}$
3. $div\,\vec{E} = 0$
4. $div\,\vec{H} = 0,$

where

$$\vec{E} = \vec{E}(z,t) = \{E_x(z,t), 0,0\} = \{E_0 \sin(\omega_1(t-z)), 0,0\} \text{ and}$$

$$\vec{H} = \vec{H}(z,t) = \{0, H_y(z,t), 0\} = \{0, H_0 \sin(\omega_1(t-z)), 0\}$$

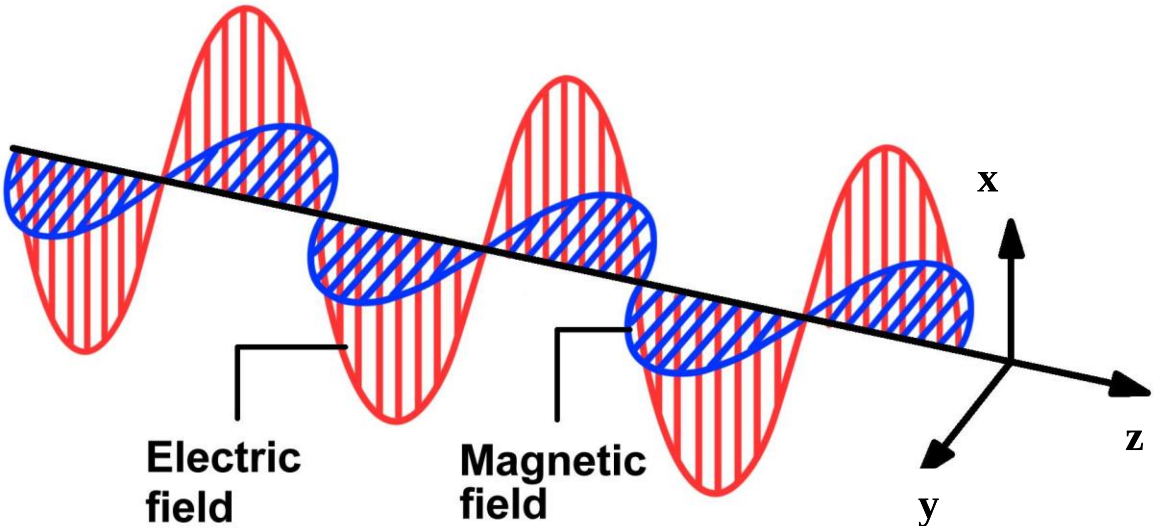
with amplitudes $E_0 = H_0 = \frac{\omega_1}{f}(g(p_1) - p_0)$ *and frequency* $\omega_1 = ((1+\alpha)\cdot f \cdot \dot{V}_0)^{1/2}$.

Proof. First, let *z* be the value of the coordinate denoting the participation phase of the harmonic price motion with respect to pre-event at time $t = t_{event}$. From Definition 5, Assumption 1** and the resulting harmonic price motion in Theorem 3, it follows that

$$H_y(z,t) = O_{D/S}^{Cash}(p) = \frac{\dot{p}}{f} = \frac{\Delta\dot{p}}{f} = \frac{\omega_1}{f}(g(p_1) - p_0)\cdot \sin\big(\omega_1 \cdot (t-z)\big) = H_0 \cdot \sin(\omega_1(t-z)).$$

This equation still holds, also if *z* denotes the value of the coordinate of any participation phase in the harmonic price motion of Theorem 3. Based on this, the results of Theorem 4 are then direct consequences from Definition 4, Definition 5 and the findings of Theorem 3. In particular, Maxwell equations follow by direct algebraic verification. ■

The appearance of the electromagnetic wave for a fixed time *t* is illustrated in the following diagram:



Total net value of securities for transaction, collected from market participants

Net demand/supply overhang in central securities store

Diagram 4.

Basic consequences of Theorem 4 are drawn as a corollary in the next lemma.

Lemma 3. Price oscillations and trading away from equilibrium.

Given are assumptions of Theorem 4. Then, in natural and partially informed markets emerge price oscillations and trading away from equilibrium. These motions can be described by Maxwell's equations in an appropriate spacetime (x, y, z, t). Oscillations are caused by pre-information events, as the one of Example 1. If there is no friction or interference with other pre-information, harmonic (price) oscillation of electromagnetism will not break down or decrease in amplitude.

Proof. From Theorem 4 it follows that, e.g. magnetic field $H_y(z,t)$ follows a harmonic motion $H_0 \sin(\omega_1(t-z))$. By Definition 5 we have $H_y(z,t) = O_{D/S}^{Cash}(p)$. We recall Assumption 1**: $\dot{p} = f \cdot O_{D/S}^{Cash}(p)$.This implies that both, overhang $O_{D/S}^{Cash}(p)$ and price *p,* follow a harmonically oscillating motion with constant amplitude. ■

6. Introducing price for information: the profit and loss of a pure value-strategy

Let us come back to the analysis of the economic implications value traders have on natural markets. By definition, a value trader accumulates

$$\dot{V}_{value}(p) = \alpha \cdot \dot{V}_0 \cdot \frac{(h(p_1) - p)}{p}$$

of stock per unit of time according to Assumption 2. But, a value trader has no intention to accumulate stock over longer periods of time. She rather wants to make sure that her trading strategy sells (buys back) every position bought (sold short) within decent time, resulting, at least on average, in a positive cash profit. Hence the accumulation of stock over time should be bounded around zero (see also Definition 3), which means that the integral

$$V_{acc,value} = \int_{t_{event}}^{t} \dot{V}_{value} dt$$

has no growth trend away from zero. In order to avoid getting lost into mathematical details, we just summarize the main result in a theorem and only list some of the important steps leading to the complete proof.

Theorem 5. Impact of pure value trading in natural market.

Let a pre-event, such as a sudden savings shock of Example 1, occur in natural market. Let, in addition, value traders with aggregate crowding-out power α join the market, being aware of the theoretical new equilibrium price p_1. Then there is a value $h(p_1) < p_1$, which value trader use as calculatory fair value, for their investment strategy as expressed by the value code in Definition 3, in order not to accumulate stock over time.

If there is no friction or interference with other pre-information, aggregate profit of value traders average out at

$$C_{value,average}^{P\&L} = \dot{V}_0 \cdot \left(p_1 - g(p_1)\right) \cdot (t - t_{event}),$$

with $g(p_1) = \frac{p_1 + \alpha h(p_1)}{1+\alpha} < p_1$ *being the real new equilibrium price.*

Proof (outline): In the first step use definitions of $V_{acc,value}$ and $\dot{V}_{value}(p)$ to see, after some calculus[3], that

$$V_{acc,value} = \alpha \dot{V}_0 \cdot (t + \frac{2(a-b)}{\omega_1 \sqrt{b^2-1}} \arctan\left(\frac{b-1}{\sqrt{b^2-1}} \tan\left(\omega_1 \frac{t}{2}\right)\right)),$$

with $a := \frac{g(p_1) - h(p_1)}{p_o - g(p_1)}$ and $b := \frac{g(p_1)}{p_0 - g(p_{1)}} > 1$. Then, in the next step we use identity

$$\arctan(r \cdot \tan(x)) = sgn\,(r) \cdot x + B(x), \qquad r\ real, |B(x)| < B\ bounded,$$

in order to derive the resulting term

$$V_{acc,value} = \alpha \cdot \dot{V}_0 \left(\frac{\sqrt{b^2-1} + b - a}{\sqrt{b^2-1}}\right) \cdot t + B(t).$$

The term $V_{acc,value}$ has no trend away from zero if and only if $\sqrt{b^2-1} + b - a$ vanishes, which is fulfilled for some value $h(p_1) < p_1$. This completes the first part of the proof. For the second part we start with the definition of cash profit and loss of value traders:

$$C_{value}^{P\&L} = -\int_{t_{event}}^{t} p\, \dot{V}_{value} dt \quad .$$

With some algebra and calculus we derive identity

$$C_{value}^{P\&L} = (g(p_1) - h(p_1)) \cdot \alpha \cdot \dot{V}_0 \cdot (t - t_{event}) + \frac{\alpha \cdot \dot{V}}{\omega_1} \sin\left(\omega_1 (t - t_{event})\right) \cdot (p_0 - g(p_1)).$$

As one can see immediately, the second term is cyclical and symmetrically bounded around zero, with the consequence that this term averages out to zero. The first term can easily be transformed into the desired identity for $C_{value,average}^{P\&L}$. ■

The fact that $C_{value,average}^{P\&L} = \dot{V}_0 \cdot \left(p_1 - g(p_1)\right) \cdot (t - t_{event})$ can also be seen from another point of view: Because $\dot{V}_0$is the constant stock supply by natural investors and $\dot{C}_1 = \dot{V}_0 \cdot p_1$ the new savings rate of natural investors after the savings shock, this increased demand would give rise to an average new equilibrium price $p_1 > p_0$. Since the true average equilibrium price is $g(p_1) < p_1$it becomes clear that the average difference $\dot{V}_0 \cdot \left(p_1 - g(p_1)\right) \cdot (t - t_{event})$ has to be cashed out of the natural market as profit to the value traders.

This is the price for information that natural investors pay on average to value traders for their "service" of accelerating price movement towards new equilibrium in partially informed market.

After quantifying the cost value traders impose to natural investors we also would like to quantify their benefit:
If a savings shock, as described in Example 1, occurs at time $t = t_{event}$ then it follows from Theorem 2 that it lasts time $T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{\sqrt{f \cdot \dot{V}_0}}$ until price *p* equals new equilibrium p_1 for the first time after $t = t_{event}$ in a pure natural market. In a partially informed market, with value traders joining, this number speeds up to $T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{\sqrt{(1+\alpha) \cdot f \cdot \dot{V}_0}}$, the time span after which new real equilibrium price $g(p_1)$ is reached. Thus, the benefit value traders have for natural traders, is that they accelerate, here in the case of Example 1, realization of intended consumption by the time difference $T_0 - T_1$. We can try to translate this benefit into a, risk neutral, interest rate r_{time}, by e.g. the following equation

$$C_1 \cdot (1 + r_{time})^{(T_0 - T_1)} = C_0,$$

Where $C_0 = \int_{t_{event}}^{T_0} \dot{V}_0 \cdot p\, dt$ and $C_1 = \int_{t_{event}}^{T_1} \dot{V}_0 \cdot p\, dt - \int_{t_{event}}^{T_1} \alpha \cdot \dot{V}_0 \cdot (h(p_1) - p) dt$ are the respective cash proceeds for consumption for natural investors, realized from t_{event} to first time where fair value is reached. In this sense, r_{time} is the price for time in a risk neutral world. We can also define a respective price for information in a risk neutral sense. If regulators require an amount of $C_{working,value}$as working capital for (aggregate) value trading then price for information r_{inform} can be defined as the interest rate equaling following equation

$$C_{working,value}\{\left(1 + r_{inform}\right)^{\Delta T} - 1\} = C_{value,average}^{P\&L}\,(\Delta T).$$

Theorem 6. Price for information and price for time.

In a risk neutral world price for information is fair if an only if price for information equals price for time:

$$r_{inform} = r_{time}.$$

Proof. See steps above. ■

7. Outline of a realistic capital markets theory (RCMT) based on Maxwell's formalism of electromagnetism: the principle of superposition of pre-information

In the last sections we gave an introduction into a new kind of thinking about how information is processed in financial markets and how this relates to returns and prices for information and prices for time. We did not yet introduce the notion of returns as "prices for risk". What follows is an agenda, a series of conjectures which have to be proven and scrutinized yet, for the purpose how these ideas could be completed into a "realistic capital markets theory", into a RCMT.
We suggest that the crossover from a risk neutral world (here defined as a world where only the notion of price for information and price for time exists) to a risky market environment shall be made in two steps.
Risk, in the idealized world of the positive savings shock of Example 1, may, in the first place intuitively, mean "change" of underlying fundamentals as the shock in savings behavior of natural investors.
In a first step, this fundamental change translates into a harmonic price oscillation with amplitude $A_0 = p_1 - p_0$. Since market participants do not know the current phase of participation cycle, A_0 could be viewed as "risk" or "volatility" a single pre-information event inducing price movements of natural market.[4]
In a second step we let a series of different fundamental "changes" occur and view the price "volatility" this series induces as price risk in natural market.

Theorem (conjecture) 7. Risk in natural market. Superposition of several pre-information events.

We conjecture that a theorem can be formulated according the following lines:

1. *Let a series of n different pre-information events at different times occur.*
2. *Argue that every single pre-information event causes an electromagnetic wave to occur as in Theorem 4 (with amplitude A_i, frequency $\omega_i,\ t > t_i,\ i = 1,..n$)*[5]
3. *Use superposition principle for electromagnetic waves to linearly sum them up to a single electromagnetic wave of oscillations (e.g. oscillation of magnetic field $O_{D/S}^{Cash}(p)$).*
4. *Translate superposition of electromagnetic wave, as represented by $O_{D/S}^{Cash}(p)$, into oscillation of price movement by Assumption 1*.*
5. *Define appropriate risk measure of the resulting price oscillation.*

Note that superposition principle is only valid for electromagnetic field, e.g. oscillation of magnetic field $O_{D/S}^{Cash}(p)$, but not for oscillations of price movements themselves (to be verified through simulation and/or mathematical analysis).

The next question on our agenda is: Is the reverse also true? Given a price time series, can we, at least in principle, always decompose magnetic field $O_{D/S}^{Cash}(p)$ into a sum of *n* different single oscillations (with amplitude A_i, frequency $\omega_i,\ t > t_i,\ i = 1,..n$,using spectral decomposition?

If yes, we can formulate

Theorem (conjecture) 8. Origin of risk in natural market.

Formulate the theorem according the following these lines:

1. *Given a natural market with market mechanism as described in Definition 4.*
2. *Given a price series of natural market for a time interval $t_0 < t < T$.*
3. *Assuming market to be in constant equilibrium at price p_0 for $t < t_0$*
4. *Then there is (up to initial condition) a unique time series for demand-supply overhang $O_{D/S}^{Cash}(p)$ by Assumption 1*.*
6. *In addition, there exists a unique set of (n) pre-information events reducing the resulting oscillation in demand-supply overhang $O_{D/S}^{Cash}(p)$ in a sum of oscillations with amplitudes A_i and frequencies $\omega_i,\ t > t_i,\ i = 1,..n$.*

Next, we conjecture that similar theorems as the last two can also be formulated in partially informed markets.
The most important ingredient of an RCMT is the complete classification of the different investment strategies joining natural market.

Theorem (conjecture) 9. Three classes of investment strategies.

There are only three independend classes of investment strategies:

1. *natural buy-and-hold investing*
2. *strategies which modify frequency ω of price oscillation*
3. *strategies which modify amplitude A of price oscillation*
   or strategies which combine above three effects,

these three forming a complete set of mode of operations how information is processed in partially informed markets.

Strategies which modify frequency are e.g. value trading as described above. If value trading is successful it absorbs cash profit from natural market with the effect of lowering amplitude of price oscillation (see Theorem 3 and Theorem 5*)*. We conjecture that amplitude modification always goes in line with absorption or emission. Absorption means here, longer term absorption of natural demand-supply overhang. When a strategy absorbs a given demand-supply-overhang for a longer term time period then we conjecture, by the turning point

Lemma 2, prices will turn and amplitude will decrease. Thus, we conjecture that we can classify the two active strategies also by this absorption property, resulting in the following statements:

1. Frequency modification: strategies having a short-term crowding-out-and-later-crowding-in behavior of natural demand and supply.
   Examples: value trading, anti-cyclical momentum trading[6], noise trading
   - useful strategies accelerate time to fair value
   - successful strategies also reduce volatility (amplitude) of oscillations

2. Amplitude modification: strategies absorbing natural demand-supply-overhang for the longer term.
   Examples: long-term anti-cyclical value investing, central bank intervention
   - useful amplitude modification reduces price volatility.
   - successful amplitude modification implies at least partial knowledge of phase in participation cycle (Successful are these strategies if they are active near turning points, knowing that natural overhang is low, absorbing rest of low natural overhang.)

As a further statement we conjecture that a list of different additional properties for strategies is qualifying them to be in the frequency or amplitude-modification group. E.g. statements like:

Lemma (conjecture) 4. Mimicking strategy lemma.
For every successful frequency/amplitude modification strategy there exists a mimicking strategy not using fundamental information but only using price-impact observation of successful frequency/amplitude modification strategies.

As examples we think that the following statements could be proofed according to the **Mimicking strategy lemma:**
- Anti-cyclical momentum strategy is a mimicking strategy of value trading or of natural pre-information price oscillations.
- Pro-cyclical market timing is a mimicking strategy of anti-cyclical market timing. It uses the fact that anti-cyclical market timing increases short-term volatility[7] which, in turn, has to decrease along the direction of price movement towards fair value.

A last theorem (conjecture) in this section shall formulate our main result:

Theorem (conjecture) 10. Price for risk.
Select a time period in natural market adjacent to a period of constant equilibrium. Let pre-information events occur which, on average, give rise to a natural price increase of $r_{risk,natural}$ per unit of time. Select, in addition, a risk measure of price movements, able to measure risk of price for information (r_{inform}), risk of price for time (r_{time}) risk of price for risk ($r_{risk,natural}$). Then, on a risk adjusted basis, extended partially informed market is in "fair" equilibrium between the different investment strategies if and only if

$$r_{risk,natural} = r_{time} = r_{inform}$$

Note that the notion of "fair" equilibrium between different investment strategies has to be defined yet. If it turns out that an appropriate definition will be too difficult to find, than the equality of the three prices itself could serve as a definition; thus the theorem would become trivial, by definition.
We conclude our *section* 7 with the remark that this conjectured RCMT as outlined here is a theory for aggregate capital markets only, not for single securities. The properties of individual securities do not play any role for derivation of aggregate results. It will be interesting to extend the methods for this aggregate capital markets theory also to single security information processing, methods being based on transferred Maxwell's theory of electromagnetism.

8. Practical and theoretical implications.

The next lemmas (conjectures) are of very practical and theoretical use:

Lemma (conjecture) 5. Second chance lemma.

Given is a major fundamental change in partially informed market, such as described in Example 1. If the first chance to profit from buying far below fair value is missed out, then there is always a second buying opportunity far below (but less far below) fair value if the following two conditions are fulfilled:
1. *The fundamental shift will not be interfered by other bigger fundamental shifts*
2. *The amplitude of price movement will not be significantly reduced by natural overhang absorption such as strategic*

selling by anti-cyclical value investors.

We suggest to show that the reduction of amplitude is basically the amount of absorption of overhang reduction plus minor interfering terms. So, for the second chance, the market low will be lower than fair value by the remaining amplitude. Iterated application of the second chance lemma yields

Lemma (conjecture) 6. Decay of oscillation towards equilibrium.

Given is a major fundamental change in partially informed market, such as described in Example 1, which is not interfered with other fundamental shifts, i.e. new equilibrium remains relatively stable. Then the existence of a successful population of anti-cyclical market timers (successful modification of amplitude) ensures the continuous decay of amplitude of oscillation around new equilibrium.

Proof (outline): Iterated application of second chance lemma: Let ε be the reduction of amplitude *A* which anti-cyclic market timers cause, through absorption of natural overhang, per half cycle of price oscillation, then the optimal second chance buying opportunity is at $A_2 = (1-\varepsilon)^2 A_0$ below fair value. Iterated full cycle amplitude is $A_{2k} = (1-\varepsilon)^{2k} A_0$ converging to zero geometrically. ■

Last we would like to develop an idea why price bubbles and their bursting can occur.

Lemma (conjecture) 7. Price bubbles formation and bursting.

Given is a major fundamental change in partially informed market, such as described in Example 1. Normal price oscillations can be explained as with Theorem 3.

- *Bubbles develop if, at some point in time, frequency modifying strategies (like value trading) become amplitude modifying strategies.*
- *Bubbles burst if, at some point in time after the upper turning point, natural investors become frequency modifying strategies increasing short-term natural stock supply.*

Proof. (outline) We suggest using the following ideas for completion of a proof:

- Start with the situation of a partially informed market with natural investors and value traders as in *section* 6. Define normal price fluctuations as oscillations with amplitude and frequency as of Theorem 3.
- Define bubble as price fluctuation having amplitude significantly above the one of Theorem 3.
- Assume that (frequency modifying strategies like) value traders could be "stopped out" for some reason near the upper turning point. "Stopped out" means they have to close their positions. Since value traders are normally selling near upper turning points, they start to buy back, which means adding to natural supply-demand overhang. Value traders become amplitude modifiers causing amplitude to increase.
- We conjecture that this effect alone would not cause a faster down-movement after the turning point (only increase of amplitude, frequency would not increase but rather decrease since value traders stopped or reduced trading, abandoning the acceleration effect they have on price fluctuations)
- Define bursting of a bubble with the property that the downward movement of the price oscillation after the upper turning point goes faster than with frequency of Theorem 3.
- If we look at the modified frequency of Theorem 3: $\omega_1 = ((1+\alpha)\cdot f \cdot \dot{V}_0)^{1/2}$, then, since α does not increase (in fact it decreases) and price adaptability f stays the same, the only way how frequency can increase during down-movement is an increase in natural supply $\dot{V}_0$, which means that natural investors turn to short-term frequency modifiers.
- To complete the proof, generalize this result to general partial informed market situations. ■

It would also be interesting if we could illustrate the effects of the bubble formation and bursting lemma by some simulations of behavior of different investor groups.

9. Outlook for further investigations. Extension to single securities and to quantum mechanics.

The conjectures of last two sections have given us an idea how powerful theory transfer from electromagnetism to aggregate capital markets could be. We could reduce all possible interactions of market strategies to the threefold classification: natural buy-and hold investing, frequency modification strategies and amplitude modification strategies. By now viewed as complex phenomena, such as bubbles or trading away from equilibrium, could become easy to understand linear problems of electromagnetism. We already could summarize, in this conjectured context, a comprehensive mode of operation of aggregate information processing in capital markets.

It will be interesting to extend, as already mentioned, the same formalism of information processing also to single securities.

In addition to that, since electromagnetism is completely compatible with quantum mechanics, it will be interesting to investigate additional theory transfers from quantum mechanics to capital markets on the same basis as we have done here in the well-shaped spacetime (x, y, z, t) of Definition 4.

Can we further categorize investment strategies using properties of electromagnetism or will the use of quantum mechanics be needed to complete a list of all subcategories?

We already emphasized amplitude modification stems from absorption and emission of demand-supply-overhang. Absorption and emission, in quantum mechanics, is an activity where localization takes place, which, by the Heisenberg's uncertainty, breaks down measurability of momentum of the underlying electromagnetic wave. Can this kind of transferred Heisenberg uncertainty be used for an improvement of understanding of risk in capital markets? Absorption and emission is also a very important notion of interaction between photons and other particles. Since photons are electromagnetic waves it will be of highest interest to investigate questions like: what is the corresponding element to an electron or proton in a transferred capital markets environment? Do they correspond to codes of special investment strategies? Can the classification of particles as described by the Standard Model of particle physics be transferred to capital markets in order to finally answer above questions of classification for investment strategies?

An additional line of interesting investigation will be the transferred use of quantum mechanics as a theory of information in order to better understand further aspects of information processing in capital markets.

This article aimed to lay the foundation for theory transfer from electrodynamics and quantum mechanics to capital markets in order to enable the kind of investigations, raised by above questions, which in turn is intended to open up a hopefully fruitful branch for successful future scientific capital markets research.

[1] Here $\dot{} := \frac{\partial}{\partial t}$

[2] Note that, by Assumption 2, it follows that
$O^{Cash}_{D\,/\,S,crowded-out} := \int_{t>t_{event}} \dot{V}_{value}(p) \cdot p\; dt = \alpha \int_{t>t_{event}} \dot{V}_0 \frac{(h(p_1)-p)}{p} \cdot p\; dt = \alpha \int_{t>t_{event}} \dot{V}_0 (h(p_1) - p) dt.$

[3] In particular use identity $\int \frac{\cos(\omega t)+a}{\cos(\omega t)+b} = t + \frac{2(a-b)}{\omega\sqrt{b^2-1}} \arctan(\frac{b-1}{\sqrt{b^2-1}} \tan\left(\omega \frac{t}{2}\right)).$

[4] If we view volatility and risk in this way, there is a second benefit value traders have for natural investors. Not only do they accelerate price movement towards equilibrium with a higher frequency $\omega_1 > \omega_0$, but also do they reduce price volatility form $A_0 = p_1 - p_0$ to $A_1 = g(p_1) - p_0$. This may be an important insight for regulators who try to forbid speculation and short selling.

[5] Extend result to the fact that different pre-information may be processed by different market clearing functions operating with different price adaptabilities f_i causing a much wider spectrum for frequencies ω_i.

[6] As anti-cyclical momentum strategy we denote here a trading strategy whose exposure is linear to the second derivative of the price (moving average) with respect to time, whereas pro-cyclical momentum strategies use exposures linear to the first derivative of the price (moving average) with respect to time.

[7] Absorption of natural overhang reduces amplitude of oscillation, causing a short-term shock, but reduces long-term volatility of oscillation.

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