How Can Knowledge Structures Determine Price Fluctuations? An Inquiry into Philosophy and Finance

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Abstract

In the present paper, we introduce a new kind of thinking about how information is processed by economic agents in financial markets. Applying an extension of the classic S5 system in modal logic, epistemic logic, we try to explain the emergence of price oscillations in the market assuming an *emergentist* epistemology. Formalizing actions of agents as the so-called primitive propositions in propositional logic, we provide both mathematical and philosophical rigorous way of facing the problem concerning market cycles. Finally, we show that the present formalization will lead us to a new epistemological approach that can overcome the well-known problem of the unrealistic economic models which is the problem of the gap between the abstract (namely mathematical) nature of the models and reality (namely the collection of real data).

1 Introduction

1.1 According to scientific literature on foundation of economic theory (from Friedman (2) to Hausman (5)), the definition of 'model' has limits that are not easy to overcome. All these difficulties are due to the epistemological differences between the abstract structure of the theory (namely the model itself) and the world (namely the real economic systems). Following Morgan (10), a *model* should be a theoretical tool capable of giving us an understanding of reality. Indeed, on one hand, a model should be general enough to provide a strong intuition about a phenomenon and, on the other hand, it can not be so general as to lose the possibility to be applied in some cases and not in others. However, this is only how it *should be*.

1.2 Before addressing this problem and before proposing our own solution, we should first understand the main approaches used to build sound explanations in economics. We underline the differences between the old methods and the new approach we are going to introduce. There are two main schools of thought in epistemology. Some philosophers and economists think that, in order to create a model, the best way is to have a mathematically consistent theory only testing it with real data. This first school believes that the mathematical consistency of the theory is the necessary (and sufficient) condition to have an explanatory model. In contrast, the second school of thought, following the epistemic revolution generated by the attempt to turn economics (a soft science) in physics (a hard science), starts from the data ending up with a model deduced from the real world. This second case relies on a blind faith in the ability of using data in order to produce a good explanation of reality. Now, looking at the history of these approaches, it is not difficult to notice that, when they were applied without compromises, they failed. Epistemologically, the first approach failed because not every mathematical consistent theory can correctly explain the reality; vice versa the second approach failed because the data can only show some patterns, some frequent phenomena in the market, without any direct explanation. In both cases, there is a lack of understanding caused by a lack of (informal) intuition.

1.3 Here we come to the same question: what can we do in front of the classic methods' breakdown? To stay on the same approaches even though they are not properly working seems a common solution in the present academic world but it seems not the best one. A possible different answer will be found by our readers in this paper. We would like to introduce a third method based on the compromise of the two previous approaches. Since a good model should not completely rely on data and should not be based only on a mathematically consistent theories, a new method should build the theory on some (informal) intuitions about the reality and should end with a mathematical, formal theory consistent with those (informal) intuitions. In this framework, the expression 'intuition' is close to the idea proposed by Feyerabend (1): *the third eye of the scientist*, namely a natural capacity of finding patterns based on the experience of practitioners and technicians.

1.4 Assuming this new epistemology, we will show how a model can be constructed following this approach. Everything will be based on two simple (informal) intuitions. First of all, since classic financial theories often assume perfect information for all market participants and this is clearly counterintuitive, in this essay we will investigate a market where this assumption does not hold introducing different agents' knowledge structures, such as knowledge, ignorance and unawareness. Secondly, we rely on the intuitive evidence that there are cyclical price fluctuations. Finally, using both intuitions, we create a model in order to explain the real financial markets.

1.5 The paper is divided into six main parts. In the first part we proposed a brief introduction. In the second part we give an analytical framework of basic definitions applied in the general framework of the model. In the third part we define the agents of the model with the whole set of possible strategies. In the fourth and fifth parts, we present how these strategies can be compatible with the agents knowledge and what is the effect on the behavior of the price. Finally, in the sixth part, we summarize the same ideas.

2 From an emergentist point of view

2.1.1 As the genetist Stuart Kauffman underlined in many essays ((7),(8)), dealing with complex systems, most of the time, means also dealing with an epistemological position called *emergentism*, namely the idea that from primitive elements (an equivalent of the Russellian atomic propositions in philosophy of logic) a system can emerge with properties that are not reducible to the features of its primitive elements. These emergent properties are systemic features of the complex system which could not be predicted or explained from a classic reductionistic point of view. In the present paper we just assume this epistemological method without discussing the analytical details of this approach but building our own case study, concerning oscillations in financial market, according to this theory.

2.1.2 Let's try to clarify this theoretical position with an example. For instance, even if it is commonly accepted that our mind is composed by the mereological sum of our neurons, our mind has more emergent properties than a single neuronal cell. By analogy, even if financial market is commonly reduced to the mereological sum of his primitive components, namely the rational agents acting in the market, the financial market has more emergent properties than a single agent. As complexity science and neuroscience is approaching the mystery of our mind without simply reducing everything to the neuronal cells, we try to apply the emergentist methodology to understand fluctuations in the financial market. Thus, given that the financial market is another complex system, in order to understand exactly this emergent phenomenon, we should start to understand its components and the relation between them. Although Jarry Fodor (3) does not use the language of emergentism, he expressed this view nicely in relation with economics speaking about the so-called *immortal economist* who tries to derive economic principles from the knowledge of physics without considering the emergent phenomena. The immortal economist is famous because all his predictions fail: he cannot see the difference between the macro-level and the micro-level: he does not understand the emergentist view. Beginning with this analogy, we start to discuss the primitive elements of our complex system in order to make them interact properly in the following sections.

2.2.1.1 The first element is the *market*. Even though this market is a simplified version of the real one, it should be able to capture the necessary features of the real economic system. Our market has three basic features. First of all, the economic agents can have cash or buy the only risky asset available. The second feature is a particular *clearing process* which functions in the following way: after collecting all buy and sell orders at the current round price p_1 and including the demand-supply overhang from the last round the total demand and supply will be cleared at price p_1. Since the price p_1 may not completely clear all demand and supply, a demand-supply overhang can remain. The new clearing price p_2 for the next clearing round will increase or decrease relatively to the demand-supply overhang. Formally, we will compute the difference between p_1 and p_2 with the following formula: $\Delta p = f$ x $demand - supply\ overhang$ where the parameter f represents the price adaptability of the market. The price adaptability will cause oscillation in the behavior of the price. Finally, this market is characterized by two different equilibria and both of them show interesting properties compared to the classic definition of the static Nash equilibrium. The first one is called *momentary equilibrium*, which is an allocation such that any agent does not want to sell and buy anything. None of the agents acting in the market have any open buy and sell position. The second type of equilibrium is called *averaging equilibrium* for a period of time T. It is the price p which would clear all the collected buy and sell orders within the period T at once.

2.2.2 Given the market previously described, a lot of different *events*, market actions, take place and we define the set of events as *world*. In our model, the definition of world is perfectly consistent with the definition that we can find in the literature about modal logic: indeed, following Saul Kripkes semantic (9), a world is a set of proposition describing an event. Different worlds contain different set of propositions describing events, namely different events as well. Now, a basic event is either a buy or a sell offer or a single market transaction while an aggregate event is the sum of two or more basic events.

2.2.3 According to our intuition, in the market, not all the events are easy to interpret. Sometimes the information we gain looking at financial dynamics is not completely clear. It is noisy. In our model, we take this into account by introducing noise into our pricing process. We assume that the market clearing process as described in section 2.2.1.1 explains the market price p_t only to a precision

ϵ. For detailed definition see section 7.5.

2.3 Proposing again Kauffman's analogy, all these components, as the neuronal cells, should form the structure of the financial market, namely the mind. However, as the mind emerges not just from the neuronal cells but from the interaction between them, in the same way the parts of our model should interact in order to make the financial market oscillations emerge. This is exactly the missing point in the epistemological approach proposed by the Fodor's *immortal economist*. Now, the question is: how can we make all these parts interact?

3 Agents and strategies

3.1 In order to combine our atomic propositions, we add an element that helps the components to create a new phenomenon, namely a phenomenon with not-reducible features (in our case, the behavior of the price). This sort of catalyst is represented by the investment strategies used by the agents. Indeed, the events, previously defined as market actions, are generated by the agents themselves.

3.2 There are two different kinds of investment strategies: long-term buy and hold investing (also referred to as natural investing strategy) and short-term trading.

3.2.1 Agents using the *natural investing strategy* apply the following code: if they have saved money, then they buy financial assets at any given price. On the other hand, if they need to consume some of their financial wealth, then they sell financial assets at any given price. Thus, agents using the natural investing strategy obey the no cash wallet condition: cash is either used for consumption or for an investment in the risky asset.

3.2.2 However, agents can also act as short-term traders. In general, trading strategies are characterized by the fact that stock of the risky asset is not accumulated over time: on average a trading strategy has zero exposure in the risky asset (this means, on average over time all agents obey the *no cash wallet condition*). For the purpose of separating the two kinds of investment strategies we allow agents to use short-term trading strategies through a Research Trading Institute only. This Research Trading Institute (from now on, RTI) receives the necessary information from agents requesting to trade and it has the privilege to trade at the current price in the market clearing process (i.e. all submitted orders are immediately transacted) in the name of the requesting agent. RTI has two possible strategies for each agent: the Value Trader Strategy and the Anti-cyclical Momentum Strategy (we will formally define these strategies in the Appendix).

3.2.2.1 Using the *Value Trader Strategy*, the agent is able to observe and analyze fundamental changes in the market. The agent, as an idealized informed trader, submits this information to RTI which is able to execute an optimal trading strategy using the fact that it knows the correct theoretical natural equilibrium price p_1, i.e. the correct average price of the future.

3.2.2.2 On the other hand, applying the *Anti-cyclical Momentum Strategy*, RTI will try to mirror the strategy of an informed agent just by looking at the price. It is important to underline that, in order to use this strategy, it is just necessary to mimic the behavior of traders that actually know it.

3.3.1 Few epistemological remarks are necessary. These agents are not classic agents for, at least, one main reason: our agents do not respect the rationality principle. Indeed, as Karl Popper claimed (12), classic economics is based on situational analysis considering rationality as a logical principle rather than a psychological principle. In 1932, Robbins with Nature and Significance of Economic Science (13) fully ensconced in the minds of economists the idea that their science is about the logic of utility maximization rather than the psychology of utility. The behavior of economic agents just reflects the solution to a logical problem of allocation. The leap between this logical problem and the mathematical problem of optimization was easily linked, for the first time, by Walras. Even though the English tradition begins with the moral sentiments of Adam Smith, the empirical Scottish tradition thought in terms of simple mathematical optimization as well. Given this background, in order to have an agent that is a perfect Turing Machine, economists added a lot of formal constrains that, most of the time, were really far from reality. For instance, the agent was always selfish, logic and, above everything, he was omniscient, namely he knew everything he needed to decide and optimize his utility.

3.3.2 This last condition is not holding in our model. Looking at the difference between the Value Trader Strategy (where the agent knows everything he needs to optimize his utility) and the Anti-cyclical Momentum Strategy (where the agent knows something but not everything) it is clear that, in our model, sometimes the agent knows everything and sometimes not. In short, we introduce an agent that can have full rationality or bounded rationality. (However, in this model, we do not consider the idea that an agent can have inconsistent beliefs).

3.4 Before understanding the consequences of the interaction between the agents and the RTI, we face another essential topic: the knowledge structure of agents and its compatibility with strategies.

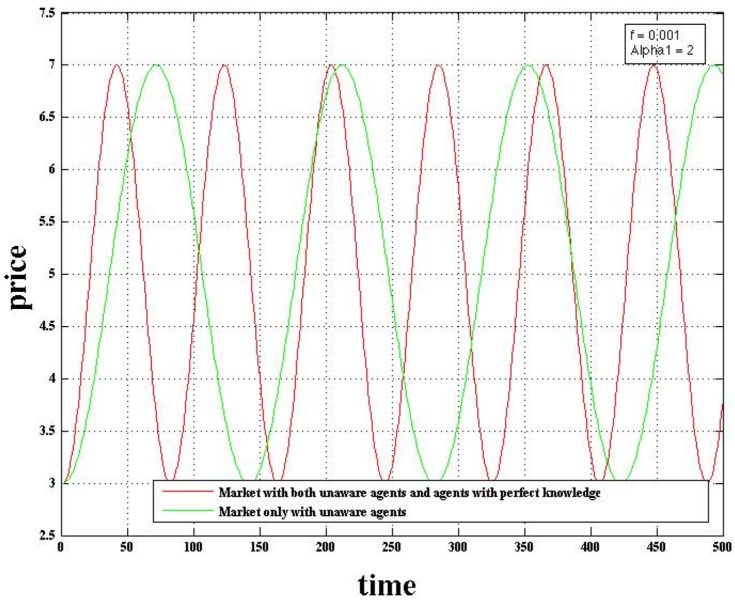

Figure 1: *Market with both unaware agents and agents with perfect knowledge. The impact of the agents with perfect knowledge is an increase of the frequency without major effect on amplitude of price oscillation.*

4 Knowledge structures

4.1.1 Each agent has a specific knowledge structure. In order to formalize this knowledge structure, we use epistemic logic, logic of knowledge and belief. It provides insights about the properties of individual knowers and it has improved our understanding of the dynamics of inquiry. It starts with the recognition of expressions like *knows that* or *believes that* and it has systematic properties that are amenable to formal study. In addition to its relevance for traditional philosophical problems, epistemic logic has many applications in computer science and economics. Examples range from robotics, network security and cryptography applications to the study of social and coalitional interactions of various kinds. Modern treatments of the logic of knowledge and belief grow out of the work of a number of philosophers and logicians writing from 1948 through the 1950s. Following Halpern (4), Rudolf Carnap, Jerzy Los, Arthur Prior, Nicholas Rescher, G.H. von Wright and others recognized that our discourse concerning knowledge and belief exhibits systematic features that admit an axiomatic-deductive treatment. Among the many important papers that appeared in the 1950s, von Wright's seminal work is widely recognized as having initiated the formal study of epistemic logic as we know it today. Von Wright's insights were extended by Jaakko Hintikka in his book Knowledge and Belief: An

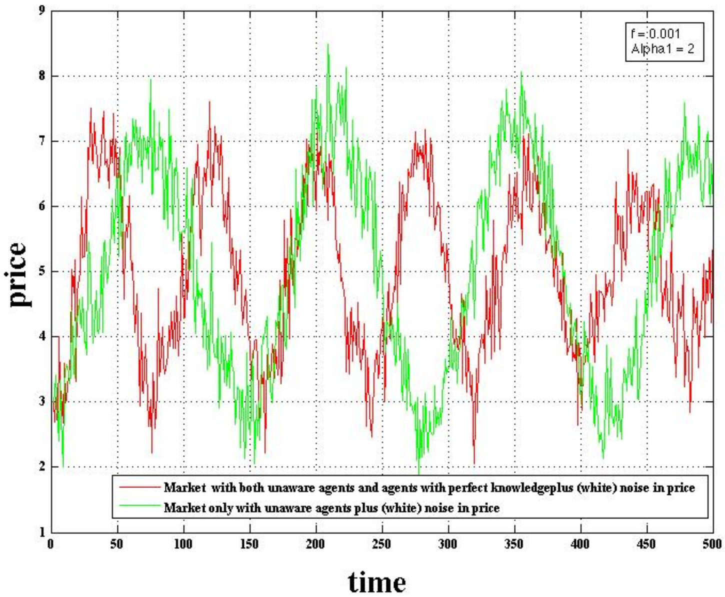

Figure 2: *Market with both unaware agents and agents with perfect knowledge. Also in the presence of noise, the impact of the agents with perfect knowledge is an increase of the frequency without major effect on amplitude of price oscillation.*

Introduction to the Logic of the Two Notions (4). In the 1980s and 1990s, epistemic logicians focused on the logical properties of systems containing groups of knowers and later still on the epistemic features of so-called "multi-modal" contexts.

4.1.2 Although epistemology has a long and honorable tradition in pure philosophy, starting with the Greeks, the idea of a formal logical analysis of reasoning about knowledge is more recent, going back to at least von Wright (4). However, only in the last ten years, it has appeared the possible-worlds analysis. The intuitive idea behind this approach is that besides the true states of affairs, there are a number of other possible states of affairs, or worlds. Given a current information, an agent may not be able to tell which of a number of possible worlds describes the actual state of affairs. An agent is then said to know a fact Φ if Φ is true in all the worlds he considers possible. Of course, to make this idea more precise, we first need a language that allows us to express notions of knowledge in the most exact way. We will use the language of modal logic in the S5 system.

4.2 Formally, we are dealing with a group of n agents. We assume that these agents think about a world that they can describe in terms of a non-empty set Φ of primitive propositions, such as 'it is sunny' or 'there was a financial bubble two years ago'. We label these primitive propositions p, z, s etc. In order to express a statement as the agent 1 knows p we should produce an augmentum of the language by modal operators $K_1..K_n$. A well-formulated formula as $K_n\Phi$ is read as the agent n knows Φ. Syntactically, a formal language is just a set of formula: the set of formula of interest to us is composed by the primitive propositions in Φ and by the more complicated well-formulated formula closing off under negation, conjunction, disjunction and the modal operator K. Semantically, the formal language applied in epistemic logic uses the idea of possible worlds and Kripke structure, namely a simple tuple $(S, \Pi, \chi_1..\chi_n)$ where S is the set of possible states for possible worlds, is an interpretation which associates each state in S with a truth assignment to the primitive propositions (for example, $\pi(s) \in (TRUE, FALSE)$ for each state $s \in S$ and each primitive proposition p). Finally, χ is an equivalence relation on S. Intuitively, $(s,t) \in \chi_1$ if agent 1 can not distinguish between state s from state t, so that if s is the actual state of the world, agent 1 would consider t a possible state of the world. Exactly because our agents are not-classic but respect the transitivity rule, namely they cannot have inconsistent beliefs, the present formal language cannot be reconciled with the idea of paradoxes, namely it is unable to model agents believing in or knowing paradoxes and contradictory statement.

4.3 In our model, the agents can have three different knowledge structures. (A) Agents can have a perfect knowledge about the average future equilibrium price after the shock in the market as described in 5.2.1. Formally, we will express this knowledge state with the following well formulated formula: $K_i p$. Given a set of primitive propositions closed under negation, conjunction, disjunction and modal operator K, knowledge is every well formulated formula anticipated by K_i. (B) The second possible knowledge structure is the structure representing awareness. Given a set of primitive propositions closed under negation, conjunction and modal operator K, $\neg K_i p \wedge K_i \neg(K_i p)$ means that an agent i is aware of p if and only if the agent i does not know p but he knows that he does not know p. Finally, the agents can be unaware. Given a set of primitive propositions closed under negation, conjunction, disjunction and modal operators K, $\neg(K_i p) \wedge \neg(K_i \neg(K_i p))$ means that an agent i does not know p and he does not know that he does not know p.

4.4 Before continuing with the last section, the second knowledge structure needs a brief specification. In the classic literature, awareness is simply expressed as the negation of unawareness, that is $\neg(\neg(K_i p) \wedge \neg(K_i \neg(K_i p)))$. This formula is equivalent to $(K_i p) \vee K_i \neg(K_i p)$. Since the first part of the previous formula is trivial because it represents only the agent with perfect knowledge, we will define awareness using only the second part of the formula, $K_i \neg(K_i p)$.

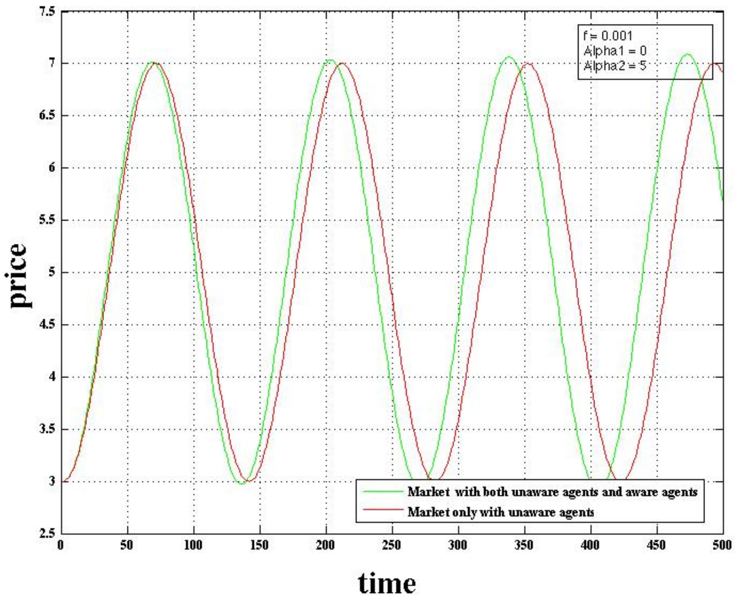

Figure 3: *Market with both unaware agents and aware agents. Aware agents have the same effect as agents with perfect knowledge: an increase of the frequency without major effect on amplitude of price oscillation.*

5 Decision trees

5.1 Given these three different knowledge structures, we can easily describe the behavior of agents. If an agent has complete knowledge of the new price equilibrium p_1, he will ask the RTI to invest his money consistently with the information that he owns. Thus, the agent will ask to the RTI to trade his money and the RTI will apply the Value Trader Strategy given his knowledge about p_1. On the other hand, if an agent is aware of the new price equilibrium p_1, then the agent will ask to the RTI to trade his money being aware of a new price equilibrium p_1 and then the RTI will apply the Anti-Cyclical Momentum Strategy: indeed, this is the strategy that request at least to be aware of a possible new price equilibrium. Finally, if the agent is not even aware of a new possible equilibrium, then the agent will not even ask the RTI.

5.2.1 Now, let's see what happen in the market if, for instance, a positive savings shock occurs. In this case, there is an external shock in the economy (e.g. it can be an unexpected decrease in the inflation rate) that will increase the money in the wallet of the

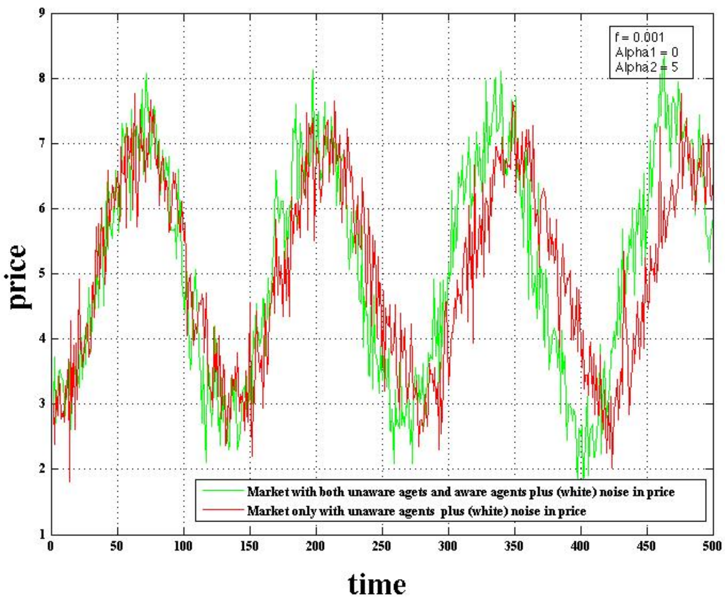

Figure 4: *Market with both unaware agents and aware agents plus noise. Again aware agents have the same effect as agents with perfect knowledge:an increase of the frequency without major effect on amplitude of price oscillation.*

agents. Since all agents, on average over time, are subject to the *no cash in the wallet condition*, they will buy financial assets increasing the demand on the market. Therefore, on average over time, more money has to be invested per unit of time giving rise to a higher average equilibrium price for the period $T = [0, \infty[$. If we assume that all the agents have the same knowledge structure and they are unaware about the future price equilibrium, then they will not even ask the RTI to invest their money but, having an unexpected surplus of liquidity, they will buy some financial assets. As we can see in Figure 1, this shock will generate harmonic oscillation in the price.The positive savings shock will increase the price but, since in the market there are imperfections (as we can recall from the unknown variable f in the equation describing the price behavior at section 2.2.1.1), the overhang of money that is generated by the increase of liquidity, will not be absorbed immediately. Thus, we will reach the two previously mentioned equilibria: one equilibrium (the so-called momentary equilibrium) that corresponds to an allocation such that any agent does not want to sell or buy anything which is reached at the extreme points of the oscillation and a second equilibrium, the new p_1 (the so-called averaging equilibrium for a period of time $T = [0, \infty[)$ where the price p_1 would clear all the collected buy and sell orders within the period T at once. This oscillation, generated by the natural investors and without any contributions of other

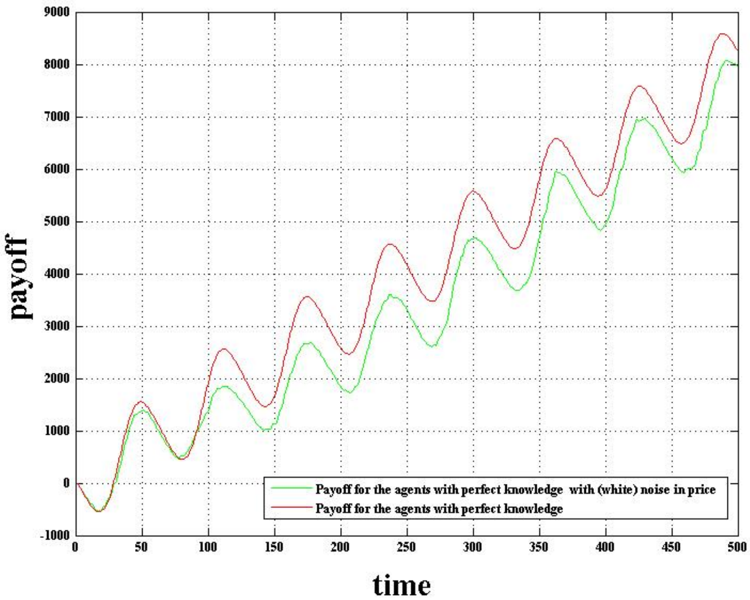

Figure 5: *Payoff for the agents with perfect knowledge with and without noise. Payoff is robust in the presence of price noise.*

strategies from the RTI, will already make the fluctuations emerge from the market. The fluctuations show clearly that the present market is an imperfect market.

5.2.2 If other strategies are added in our simulation environment, we easily see that the predominance of one knowledge structure compared to another influences the behavior of the price. In the case of knowledge, the curve will increase the frequency and slight decrease in the amplitude (Figure 2 and Figure 3). In the case where the majority of agents are aware, the curve of the price will increase its frequency and with no major effect on the amplitude compared to the behavior of the price when the agents are unaware (Figure 4 and Figure 5).

5.3 Moreover, if we try to compute the payoff of each strategy, we see that the payoff is always positive in the case of perfect knowledge about the future averaging equilibrium price but, in the case of awareness, the payoff is positive without noise but negative with noise (for a detailed definition of noise see section 7.5). In other words, if the market is without any disturbance, the two strategies are both positive: the awareness knowledge structure gives the agent less money on the long run. On the other hand, in a market with noise, the perfect knowledge structure is always more profitable.

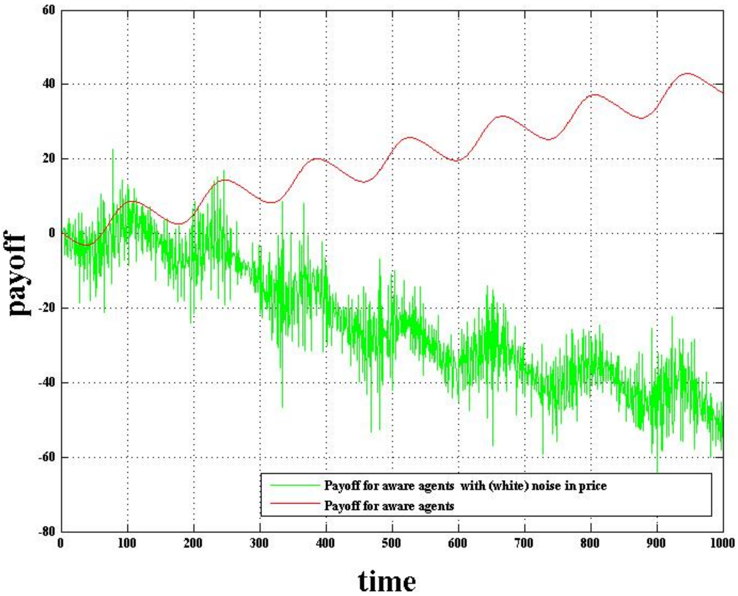

Figure 6: *Payoff for aware agents with and without noise. Payoff deteriorates significantly in the presence of price noise.*

6 Conclusion

6.1 To conclude, the present model is able to provide two important considerations. First of all, developing the analogy between the mind and the market, we finally have a model to understand the emergence of market oscillations from some given elementary propositions called agents or, more precisely, their actions. The second consideration is more related with the epistemological foundation of our model: we tried to overcome the distinction between world and reality. From this point of view, our model is presenting a new way of creating models that is not simply the collection of data (the picture of the world) or the mathematical abstraction (the theory). This article, following Wittgensteins claim, believes that such a simple and intuitive approach can lead us to a new epistemology and a fruitful explanation of the world.

6.2 This simple model can explain much more complicated (or even complex) phenomena because, as the author of the *Tractatus Logico-Philosophicus* wrote: *Simplex Sigillum Veri* (13). Most of the time, simplicity is the seal of truth.

7 Appendix

7.1 All the simulations that we have presented in this research paper were coded and tested by the authors. The authors have used the program MatLab. The MatLab syntax is built around the MatLab language involving typing MatLab codes and executing text files containing functions. The version used in the present work is MatLab 7.1.

7.2 The simulations concerning the positive savings shock are all based on the same parameters: the initial price is fixed on an arbitrary value 3, the positive shock is a shock giving rise for a price increase of 2 in the new (average) equilibrium price and the result is an harmonic oscillation. Since the equation describing the behavior of the price is Δp = f x $demand - supply\ overhang$, we decided to set the value of f, namely the price adaptability as a constant for all the simulations proposed in the present paper the estimated value is 0.001. In the code, we have simulated the impact on the behavior of the price of two strategies: the value trader strategy and the anti-cyclical momentum strategy. For each strategy we have decided a specific value of Alpha that is able to give the weight and dimension of the impact of each strategy (Alpha 1 for value trader strategy and Alpha 2 for anti-cyclical momentum strategy). Alpha 1 has a value of 2 because in the market there is just the value trader strategy . Alpha 2 = 5 and Alpha 1= 0: in this case, only anti-cyclical momentum strategy has an actual impact in the market.

7.3 *Value Trader Strategy*: this strategy is applicable only for (idealized) investors that (a) hold security only short-term, that (b) is informed about the correct averaging equilibrium for the period $T = [0, \infty[$ and that (c) the trading behavior can be formally described with the following equation $\boxed{V_{volume}(p) = \alpha_1 V_0 \frac{(p_1 - p)}{p}}$. Alpha is the parameter introduced in the second point of the present Appendix, V0 is the initial trading volume of money in the market, p is the price in equilibrium and p1 is the actual price. This equation leads to an informal (but extremely clear) code. When RTI applies this strategy will apply the following code:

- observe price p in the market at time t
- submit buy and sell order in order to ensure purchase or sales of the following number of securities:
$V_{volume}(p) = \alpha_1 V_0 \frac{(p_1 - p)}{p}$
- end of the code.

7.4 *Anti-cyclical Momentum strategy*: this strategy is applicable only for investors that (a) know that they do not know the theoretical natural equilibrium price and that (b) want to mirror the action of the Value Traders Strategy. Formally, $\boxed{V_{volume}(p) = \alpha_2 V_0 (p_t - 2p_{t-1} + p_{t-2})}$. When RTI applies this strategy will apply the following code:

- observe the second derivative of the price without knowing the theoretical natural equilibrium price
- submit buy and sell order in order to ensure purchase or sales of $V_{volume}(p) = \alpha_2 V_0(p_t - 2p_{t-1} + p_{t-2})$
- end of the code

7.5 Finally, we have added (white) noise $\gamma\epsilon(t)$, where $\epsilon(t) \sim N(0,1)$, as a disturbance in the price behavior. The noise is always computed in the following way:
$\boxed{noisyp(t+1) = f(DSO) + p(t) + \gamma\epsilon(t)}$. DSO is the demand-supply overhang in the market after the shock, and gamma is the noise load.

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